The Case
for High Field Fusion (abridged)

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2017 Fusion Power Associates

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 available at firefusionpower.org

psfc.mit.edu
Case: The high magnetic field path is optimal to obtain our absolute science and energy goals

- From plasma science viewpoint there are no serious “tradeoffs” in the design of your MFE burn/energy mission, you always maximize B field strength

- Achieving high B field with electromagnets has fundamental science limits; understanding this evolving science allows us as plasma physicists how to best meet our science and energy missions
Why now?

• 15+ years since we talked about this.. many of our younger scientists don’t recall key features of debate about the “tactics” involved in achieving burning plasmas.

• And haven’t things changed meanwhile?
  ➢ In physics of plasmas, magnets, etc.
  ➢ Or maybe we just have more experience & insight
Volumetric fusion power density

\[ \beta \equiv \frac{P_{th}}{p_{magnetic}} = \frac{P_{th}}{B^2 / 2\mu_o} \]

\[ P_f \approx 8 P_{th}^2 \]

Troyon limit (tokamaks)

\[ \beta_N = \frac{\beta q}{5\epsilon S(\kappa)} \]

Generic

\[ P_f \sim \beta^2 B^4 \]

Tokamak

\[ P_f \sim \frac{\beta_N^2 \epsilon^2 S(\kappa) B^4}{q^2} \]

Troyon, Gruber Phys. Lett. 110A (1985)
Confinement: tokamak

- Expressing confinement through “wind-tunnel” dimensionless scaling laws

\[ B \tau \propto \rho_*^{3.1} \beta^0 \nu^{-0.35} q_{95}^{-1.4} \kappa^{2.2} \]

Petty

Extract \( R, B \) at Fixed \( R/a \)

\[ \tau \sim R^{3.1} B^{2.1} \]

ITPA

Luce, Petty, Cordey PPCF 50 (2008)
Energy gain at fixed physics & shape parameters

\[ \frac{P_{th}}{\tau_E} = \frac{R^2 B^2}{R_{\text{H98}}^2 B^{3.5}} \]

\[ \frac{P_{th}}{\tau_E} = \frac{R^2 B^4}{R_{\text{Petty}}^3 B^{2.1}} \]

\[ \frac{P_{th}}{\tau_E} = \frac{R^2 B^2}{R_{\text{ISS04}}^2 B^{2.2}} \]
High B (+ strong shaping) enables stationary pedestal with high absolute pressure

\[ \beta_{N,\text{Ped}} \leq \left( \frac{\Delta \psi_{\text{ped}}}{5\%} \right)^{3/4} \]

\[ \beta_{N} \sim \frac{p_{\text{ped}}}{p_{\text{magnetic}}} \frac{q}{\epsilon} \]

\[ p_{\text{ped}} \leq \left( \frac{\Delta \psi}{5\%} \right)^{3/4} \frac{B^2}{q} \]

~ Peeling-Ballooning Stability Limit

\[ B \sim 5.7 \, \text{T} \]

Comparisons of C-Mod Pedestal with EPED

Measured Pedestal Height (kPa)

EPED Predicted Max. Pedestal Height (kPa)

Hughes APS 17

Snyder et al NF 2011
### Am I happy or sad?

<table>
<thead>
<tr>
<th>Issue</th>
<th>Scaling</th>
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<tbody>
<tr>
<td>Power density</td>
<td>$B^4$</td>
<td>Density (tokamak)</td>
<td>$R^{-1} B^1$</td>
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<tr>
<td>Confinement (generic)</td>
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<td>Density (stellarator)</td>
<td>$\beta B^{2.5}$ (burning)</td>
</tr>
<tr>
<td>Confinement (tokamak)</td>
<td>$R^{2.7} B^{3.5}$ (H_{98})</td>
<td>Heat exhaust: min. $f_z$</td>
<td>$R^{1.3} B^{0.9}$</td>
</tr>
<tr>
<td></td>
<td>$R^{3.1} B^{2.1}$ (Petty)</td>
<td>Heat exhaust: $q//e$</td>
<td>$B^{-1}$ (burning)</td>
</tr>
<tr>
<td>Confinement (stellarator)</td>
<td>$R^{2.8} B^{2.1}$</td>
<td>Runaway e- amp.</td>
<td>$\exp (R^{0.28} / B^{0.3})$</td>
</tr>
<tr>
<td>Gain</td>
<td>$R^{2-3.1} B^{4-5.5}$</td>
<td>Synchrotron: runaways</td>
<td>$B^2$</td>
</tr>
<tr>
<td>Stable pedestal/I-mode</td>
<td>$~ \beta_N B^2$</td>
<td>Synchrotron:thermal</td>
<td>$\sim B^{1.5}$</td>
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<td></td>
<td>TAE</td>
<td>$n\sim B$, $v_A \sim B$</td>
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Whyte, Case for High Field Fusion, APS 2017
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<td>Confinement (tokamak)</td>
<td>$R^2.7B^{2.1}$ (Petty)</td>
<td>1998</td>
<td>2008</td>
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<td>Confinement (stellarator)</td>
<td>$R^{2.8}B^{2.1}$</td>
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<td>Gain</td>
<td>$R^{2-3.1}B^{4-5.5}$</td>
<td>2010</td>
<td>2016</td>
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<td>$B_NB^2$</td>
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<td>$400-500B$ (burning)</td>
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<td>$\exp\left(\frac{R^{0.28}}{B^{0.3}}\right)$</td>
<td>2005-17</td>
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Am I happy or sad? I’m happier than before

Whyte, Case for High Field Fusion, APS 2017
Electromagnets & Tokamak plasmas: same physics

\[ \nabla \times \vec{B} = \mu_0 \vec{j} \]
\[ \nabla p = \vec{j} \times \vec{B} \]
\[ P = \eta \, j^2 \]

Ampere’s law
Force balance
Ohmic heating

\[ B \sim j \]
\[ P_{\text{magnet}} \sim B^2 \]
As in toroidal plasma physics, aspect ratio is a critical and complex optimization.

\[
x \equiv \frac{a}{b}
\]

\[
M = \frac{2x + 1}{3(1 - x)}
\]
Simple toroidal “solenoid” to explore limits

\[ R = 4 \text{ m}, \quad A = 4, \quad B_0 = \frac{B_{\text{max}}}{2} \]

\[ B = \frac{\mu_0}{2\pi} \int j_z \pi R dR \]

\[ B_{\text{max}} = 0.3\pi j_{MA/m^2} \]

\[ x \equiv \frac{a}{b} = \frac{2}{3} \]

\[ M = 2.3 \]

\[ \sigma_{\text{max}} [\text{MPa}] \approx M \frac{B_{\text{max}}^2}{2\mu_0} \]
LN-cooled copper + steel for stress loading
Pulsed due to lack of active cooling

\begin{align*}
B_0 & \approx 11 \text{ T} \\
B_{\text{max}} & \approx 22 \text{ T}
\end{align*}
Superconductors: zero resistivity, but a restricted operating space in T, j and B
Superconductors: critical current, at fixed T, depends on SC type and B

\[ \frac{J_c}{J_{c,0}} = \left( \frac{B}{B_0} \right)^{-\alpha} \]

<table>
<thead>
<tr>
<th>SC Type</th>
<th>$J_{c0}$</th>
<th>$B_0$</th>
<th>$\alpha$</th>
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<tbody>
<tr>
<td>Nb-Ti</td>
<td>$10^3$</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Nb$_3$-Sn</td>
<td>$10^3$</td>
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$T \sim 4$ K, $B > B_0$
**铌-锡超导体**：
磁通量限制在约T~4K

 coils

- 50% SS
- 22% cool
- 25% Cu
- 3% SC

![Coil Cross-section](image)

$B\text{ max} \sim 7$ T

$B\text{ max} \sim 12$ T

$\sigma_{\text{max}}$ (MPa) vs $B_{\text{max}}$ (T)

$4\text{ K}$

$\dot{j}_{\text{crit}}$ Nb-Ti

$\dot{j}_{\text{crit}}$ Nb$_3$-Sn

SS strength limit
NAS study: Cryogenic Cu could study burning plasma science at 25x smaller volume than Nb$_3$Sn

\[ p_{th} \tau_E \sim R^{2.7} B^{5.5} \]

Volume \( \sim R^3 \sim 1/B^5 \)

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<tr>
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<tbody>
<tr>
<td>B (T)</td>
<td>10</td>
<td>5.3</td>
</tr>
<tr>
<td>R (m)</td>
<td>2.14</td>
<td>6.2</td>
</tr>
<tr>
<td>Q</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( \tau / \tau_{CR} )</td>
<td>&gt; 1</td>
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</tr>
<tr>
<td>( V_p ) (m$^3$)</td>
<td>30</td>
<td>800</td>
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25x
Tactics? High-B, compact was known to have ~10-fold performance to cost ca. 1990 but pulsed

Compact Tokamak Ignition Concepts J. Willis J. Fusion Energy 1989
High-Temperature (HTS) REBCO superconductors

\[ \frac{J_c}{J_{c0}} = \left( \frac{B}{B_0} \right)^{-\alpha} \]

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<tr>
<td>REBCO</td>
<td>2.5 \times 10^3</td>
<td>5</td>
<td>0.6</td>
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\( T \approx 4 \text{ K}, \ B > B_0 \)
With HTS magnets, stress is the only limit \( \Rightarrow \) multiple design choices to achieve \( B_{\text{max}} > 20\text{T} \)

\[
B_{\text{max}} \sim 23\text{T} \\
B_0 \sim 9.2\text{T} \\
\sigma_{\text{max}} \sim 700\text{ MPa}
\]
HTS magnets clearly change the tactical landscape for magnetic fusion

- Diversification
- Risk distribution
- Speed
Empirical Greenwald density is a disruptive limit in tokamaks

\[ n \leq n_{Gr} = \frac{I_p}{\pi a^2} \propto \frac{S(\kappa)}{q} \frac{B}{R} \]

- JET 9
Power exhaust: tokamak divertor Solutions

\[ P_{\text{rad}} \sim n_{\text{div}}^2 f_Z F(T_e) \]

\[ n_{\text{div}} \sim n_{\text{core}}^2 \]

\[ n_{\text{core}} \propto \frac{S(\kappa) B}{\epsilon R} \]

Required impurity Fraction to Detach\(^1\)

\[ c_Z \propto \frac{P_{\text{SOL}}}{B_p f_{Gr}^2} \]

Required impurity Fraction to Dissipate Psol in H-mode\(^2\)

\[ f_Z \sim B^{0.9} R^{1.3} \]

\[ \lambda_{q//} \propto \epsilon \rho_{\text{pol}} \sim 1 / B_{\text{poloidal}} \]

\[ q_{//} \propto P_{\text{SOL}} B / R \]

\(^1\)Goldston et al PPCF 2017, APS17