



Massachusetts Institute of Technology



Plasma Science & Fusion Center

The Science Case for High Field Fusion

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Nuclear Science and Engineering

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psfc.mit.edu



What are the most pressing issues in magnetic fusion?

Science mission

- **Self-heated burning plasma**

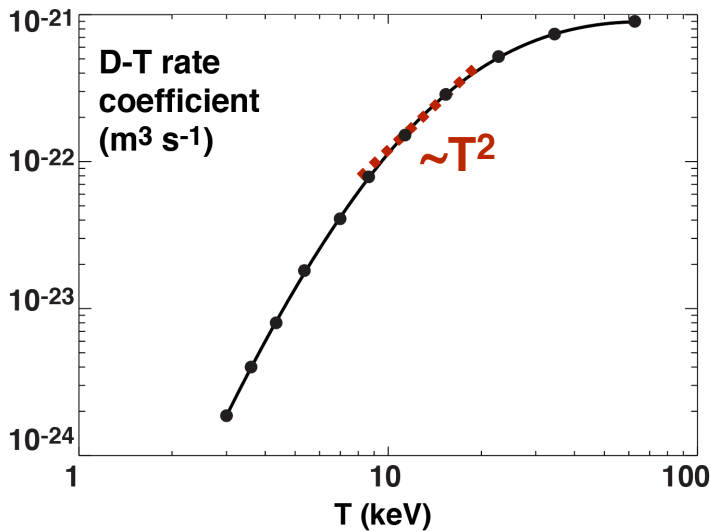
Energy mission

- **Energy gain**
- **Fusion power density**

What are the most pressing issues in magnetic fusion?

Science mission

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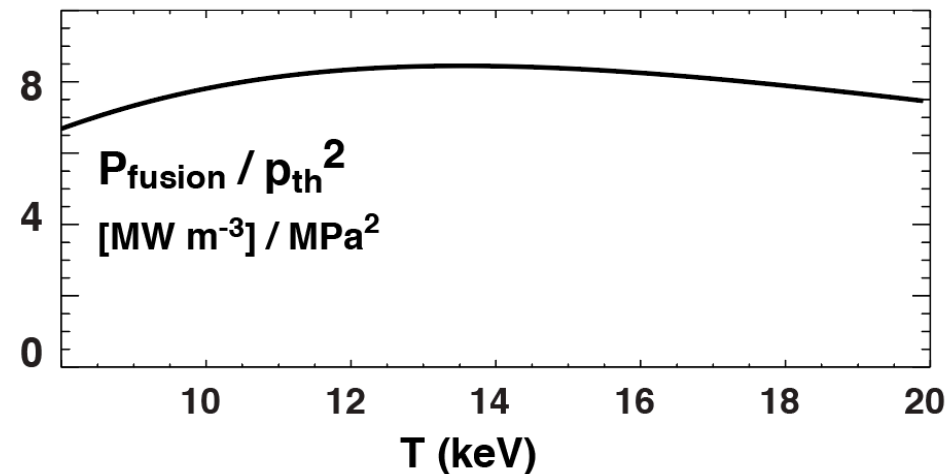


$$P_{D-T, \text{fusion}} \propto n^2 T^2$$

$$p_{th} \equiv 2nkT$$

Energy mission

- $P_f [MW m^{-3}] \simeq 8 p_{th, MPa}^2$
- Fusion power density**



What are the most pressing issues in magnetic fusion?

Science mission

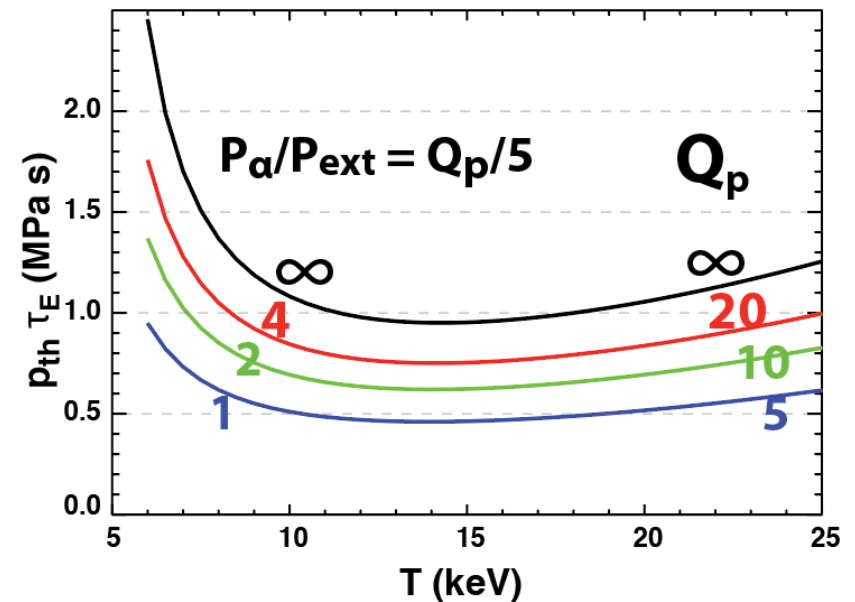
- Self-heated burning plasma

$$p_{th} \tau_E \approx 1 \text{ MPa} \cdot \text{s}$$

$$T_i \approx T_e \sim 10 - 20 \text{ keV}$$

Energy mission

- Energy gain
- Fusion power density



Most pressing needs center on achieving absolute nuclear physics & confinement; NOT R , B , β , q , etc.

Science mission

- Self-heated burning plasma

$$p_{th} \tau_E \simeq 1 \text{ MPa} \cdot s$$

$$T_i \simeq T_e \sim 10 - 20 \text{ keV}$$

Energy mission

- Energy gain
- Fusion power density

$$P_f \left[\text{MW } m^{-3} \right] \simeq 8 p_{th, \text{MPa}}^2$$

Case: The high magnetic field path is optimal to obtain our absolute science and energy goals

- **From plasma science viewpoint there are no serious “tradeoffs” in the design of your MFE burn/energy mission, you always maximize B field strength**
- **Achieving high B field with electromagnets has fundamental *science* limits; understanding this evolving science allows us as plasma physicists how to best meet our science and energy missions**

Case: The high magnetic field path is optimal to obtain our absolute science and energy goals

- From plasma science viewpoint there are no serious “tradeoffs” in the design of your MFE burn/energy mission, you always maximize B field strength
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~1/2 of review

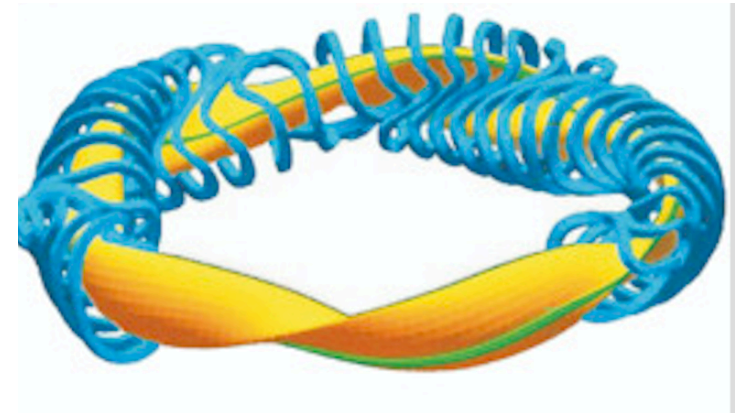
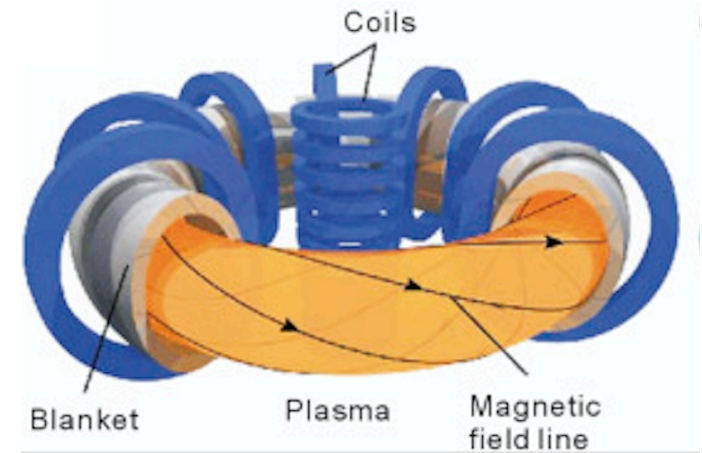
~1/2 of review

Why now?

- Wasn't all this settled with Snowmass → ITER in 2002?
- 15+ years since we talked about this.. many of our younger scientists don't recall key features of debate about the “tactics” involved in achieving burning plasmas.
- And haven't things changed meanwhile?
 - In physics of plasmas, magnets, etc.
 - Or maybe we just have more experience & insight

Ground rules

- Toroidal geometries: tokamak, stellarators, etc.
- R : major radius a : minor radius
- $\epsilon = a/R$
- If not stated explicitly, shaping is assumed fixed



Size, Field + physics dependences?

Issue	Scaling
Power density	
Confinement (generic)	
Confinement (tokamak)	
Confinement (stellarator)	
Gain	
Stable pedestal	

Issue	Scaling
Density (tokamak)	
Density (stellarator)	
Heat exhaust: min. f_z	
Heat exhaust: $q//$	
Runaway e- amp.	
Synchrotron: runaways	
Synchrotron:thermal	
TAE	

Volumetric fusion power density

$$\beta \equiv \frac{p_{th}}{p_{magnetic}} = \frac{p_{th}}{B^2 / 2\mu_o}$$

Troyon limit (tokamaks)

$$\beta_N = \frac{\beta q}{5\epsilon S(\kappa)}$$

+

$$P_f \approx 8 p_{th}^2$$



Generic

$$P_f \sim \beta^2 B^4$$

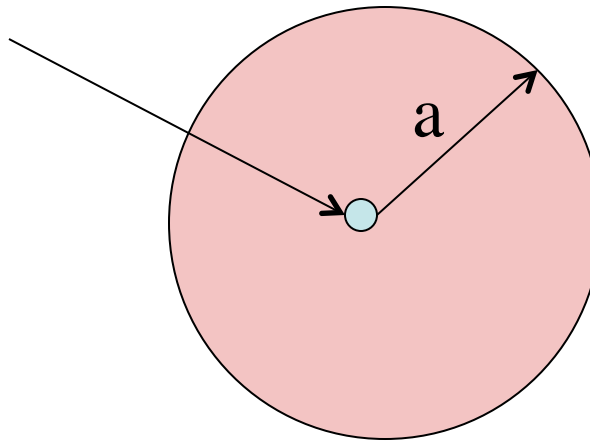
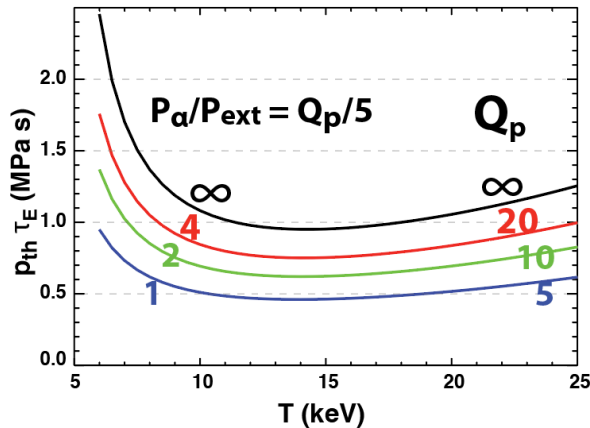
Tokamak

$$P_f \sim \frac{\beta_N^2 \epsilon^2 S(\kappa) B^4}{q^2}$$

Troyon, Gruber Phys. Lett. 110A (1985)

Confinement (generic)

$$\rho = \frac{(mT)^{1/2}}{e B} \propto \frac{1}{B}$$



Diffusive transport

$$\tau \propto \left(\frac{a}{\rho} \right)^2 \equiv \rho_*^{-2}$$

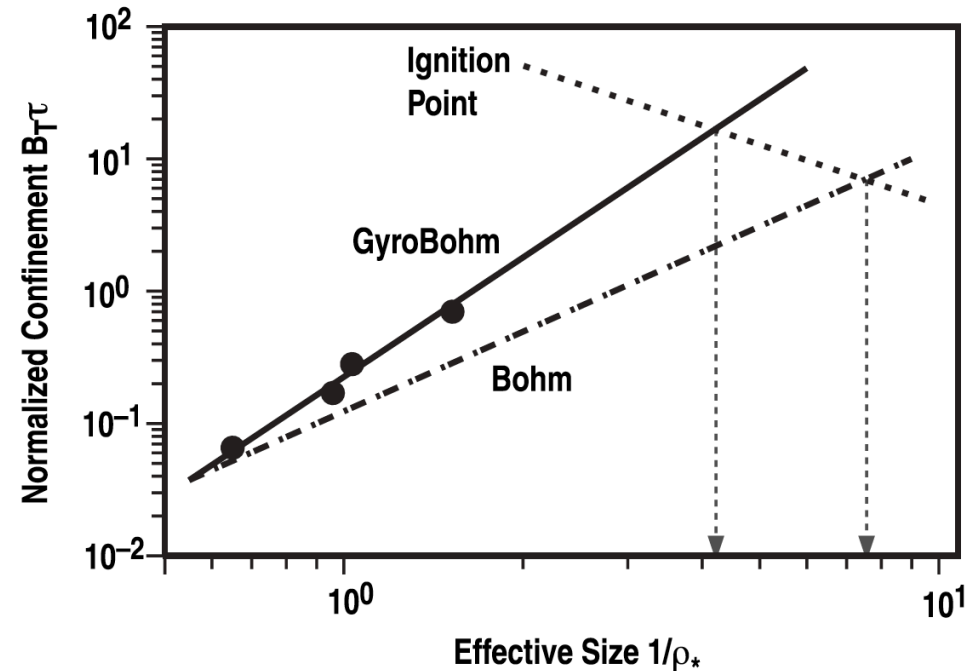
Confinement: tokamak

- Expressing confinement through “wind-tunnel” dimensionless scaling laws

$$B \tau \propto \rho_*^{-3.1} \beta^0 \nu^{-0.35} q_{95}^{-1.4} \kappa^{2.2} \quad \text{Petty}$$

Extract R, B at
Fixed R/a

$$\tau \sim R^{3.1} B^{2.1}$$



ITPA
Luce, Petty, Cordey PPCF 50 (2008)

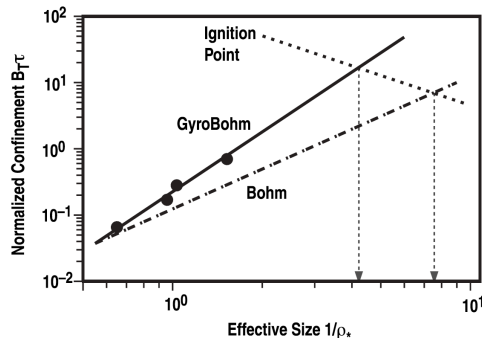
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- Empirical H-mode scaling

$$\tau_E \sim H n_{20}^{0.1} M^{0.5} I_{p,MA}^{0.85} R^{1.2} a^{0.3} \kappa^{0.5} B^{0.2} P^{-0.5}$$

Dimensionless

$$B \tau_{H_{98}} \propto \rho_*^{-2.7} \beta^{-0.9} \nu^{-0.1} q^{-3} \epsilon^{0.7} \kappa^{3.3}$$

Target p_{th}

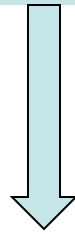
Extract R, B at
Fixed R/a

$$\tau_{H_{98}} \sim R^{2.7} B^{3.5}$$

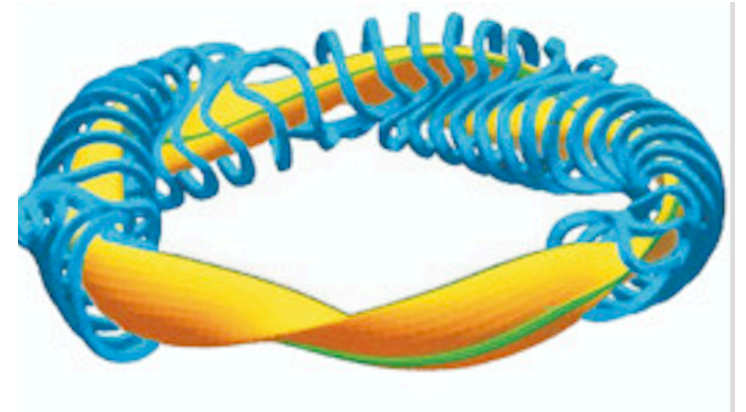
Confinement: stellarator

$$\tau_{ISS04} = \tau_{Bohm} \rho_*^{-0.8} \beta^{-0.2} \nu^{0.0}$$

*Extract R, B at
Fixed R/a & p_{th}*



$$\tau_{ISS04} \sim R^{2.8} B^{2.2}$$



Dinklage, A. et al. Fusion Sci. & Tech. 51 (2007)

Energy gain at fixed physics & shape parameters

Target

$$p_{th}$$

X

$$\tau_E$$

=

Target

$$p_{th} \tau_E$$

Generic

$$R^2 B^2$$

$$R^2 B^4$$

$$B^2$$

H₉₈

$$R^{2.7} B^{3.5}$$

$$R^{2.7} B^{5.5}$$

Petty

$$R^{3.1} B^{2.1}$$

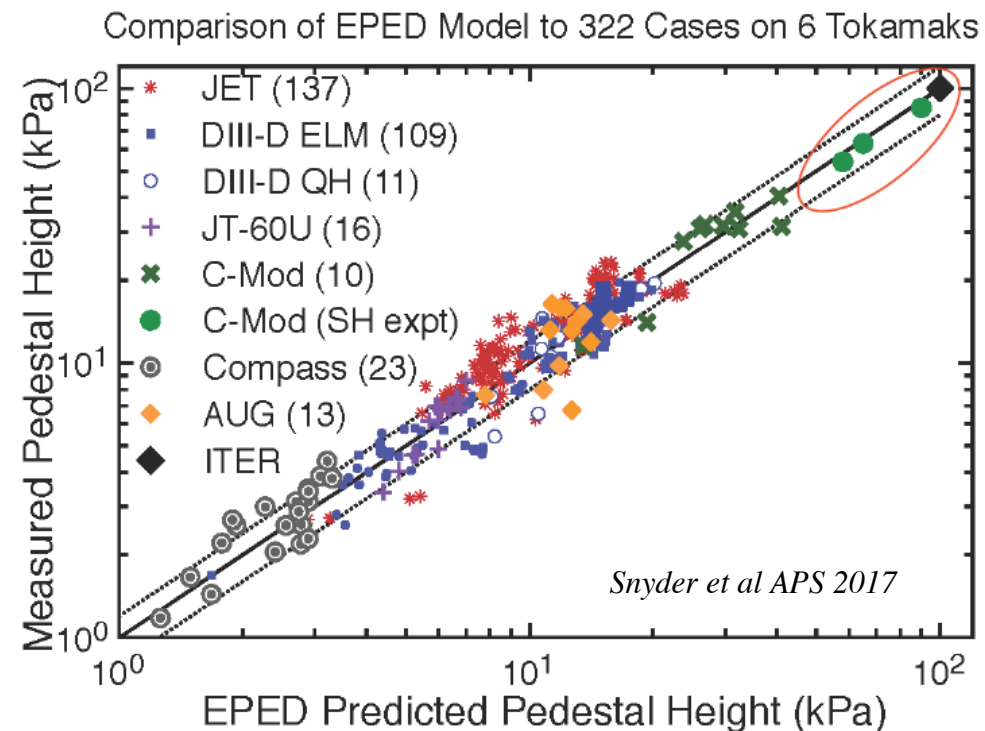
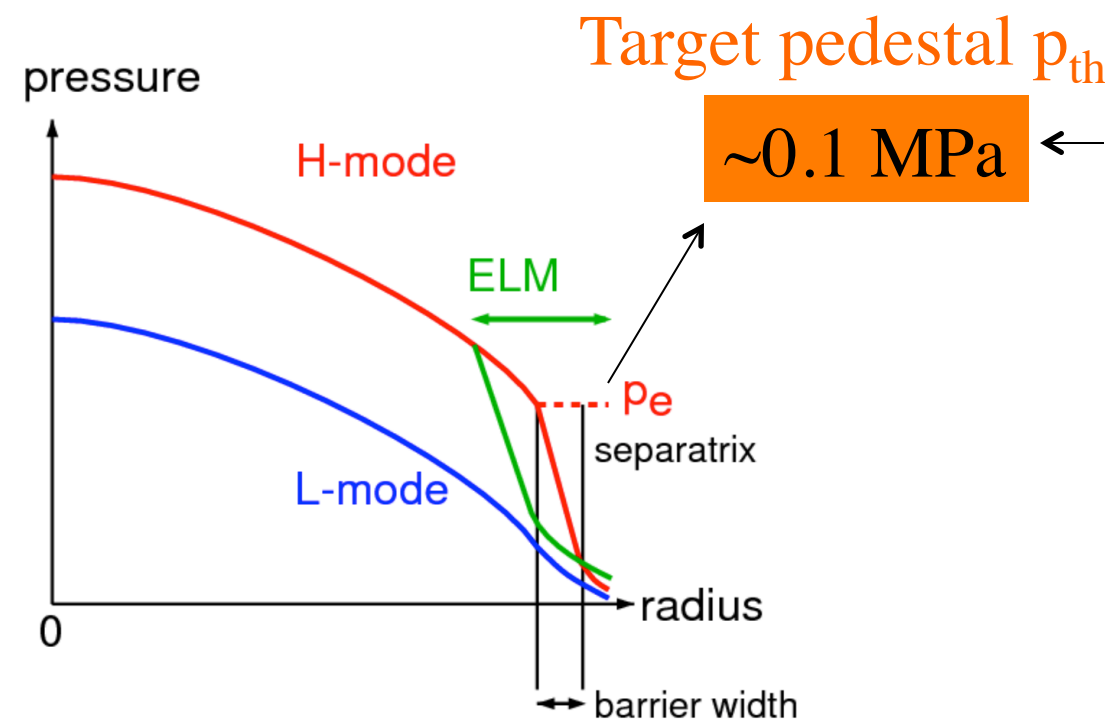
$$R^{3.1} B^{4.1}$$

ISS04

$$R^{2.8} B^{2.2}$$

$$R^{2.8} B^{4.2}$$

Tokamak boundary pedestal is required achieve high global pressure, but cannot be unstable to Edge Localized Modes



High B (+ strong shaping) enables stationary pedestal with high absolute pressure

~ Peeling-Ballooning Stability Limit

Snyder et al NF 2011

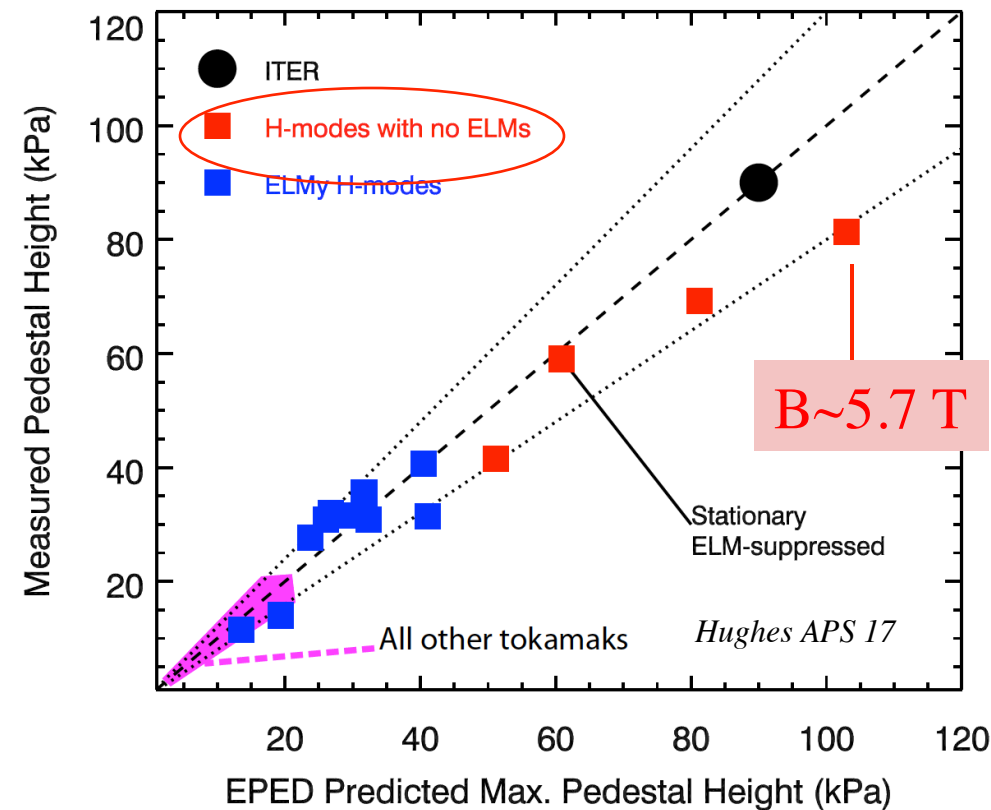
$$\beta_{N, Ped} \leq \left(\frac{\Delta\psi_{ped}}{5\%} \right)^{3/4}$$

$$\beta_N \sim \frac{p_{ped}}{p_{magnetic}} \frac{q}{\epsilon}$$

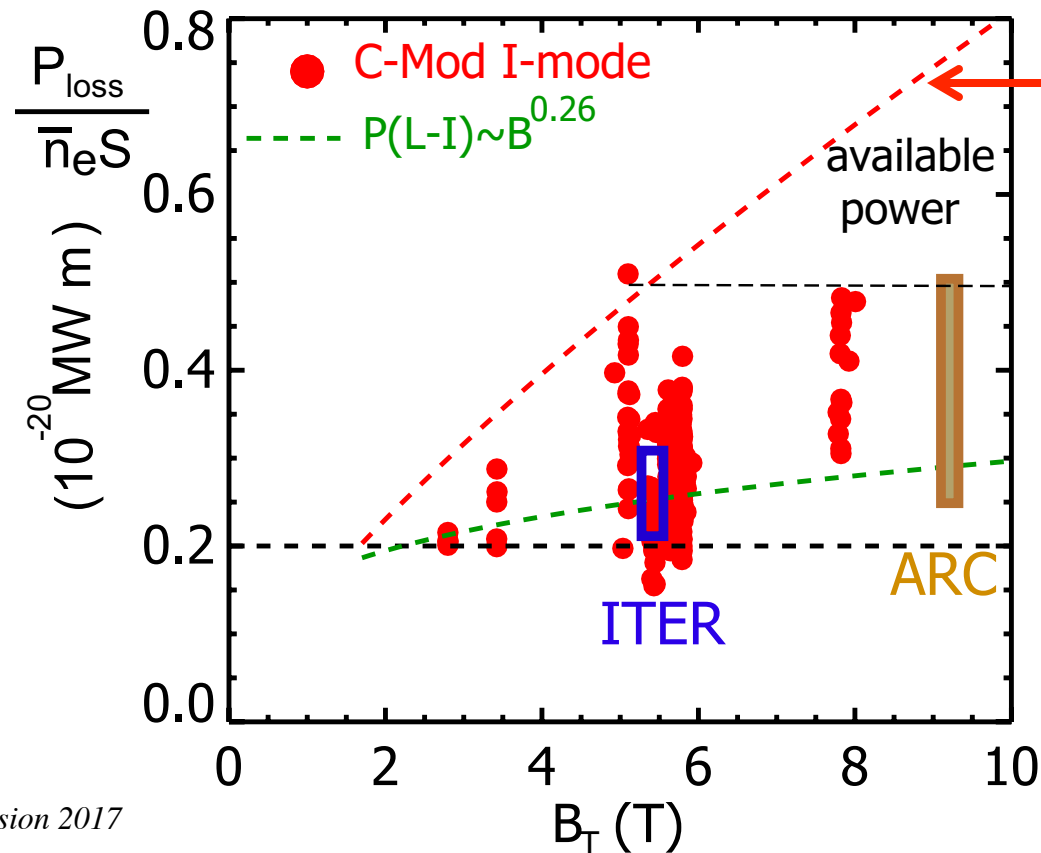


$$p_{ped} \leq \left(\frac{\Delta\psi}{5\%} \right)^{3/4} \frac{B^2}{q}$$

Comparisons of C-Mod Pedestal with EPED



High-field provides robust access to intrinsically ELM-free I-mode, in part by suppressing H-mode transitions

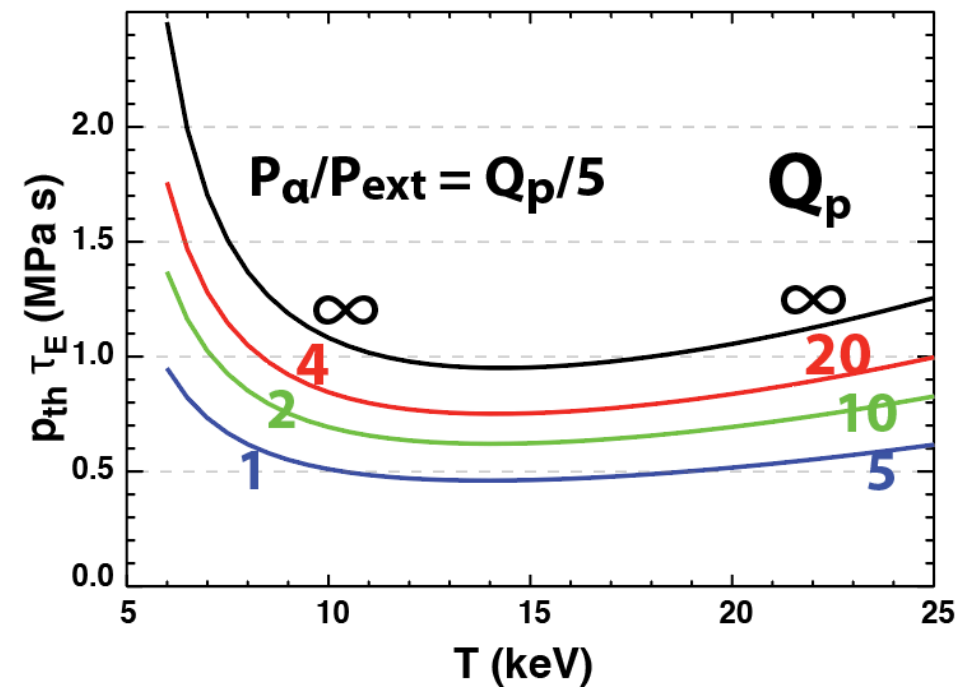


$$P_{L-H} \propto B$$

Hubbard et al Nucl. Fusion 2017

Density: tokamak

If T fixed by Lawson optimization then
 $P_f \sim n^2$
but can you get there?



M. Greenwald, *Density limits in toroidal plasmas*, Plasma Phys. Control. Fusion **44** (2002) R27-R53

Density limit in stellarator

$$n_{20} \leq 0.25 P_{tot}^{0.5} B^{0.5} a^{-1} R^{-0.5}$$

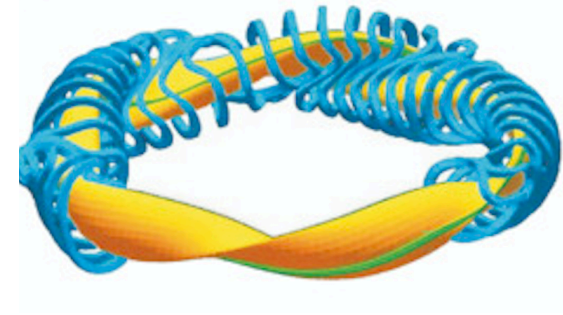
Sudo, et al. Nucl. Fusion 30 (1990)

In a burning plasma

$$P_{tot} \propto p_{th}^2 R^3$$

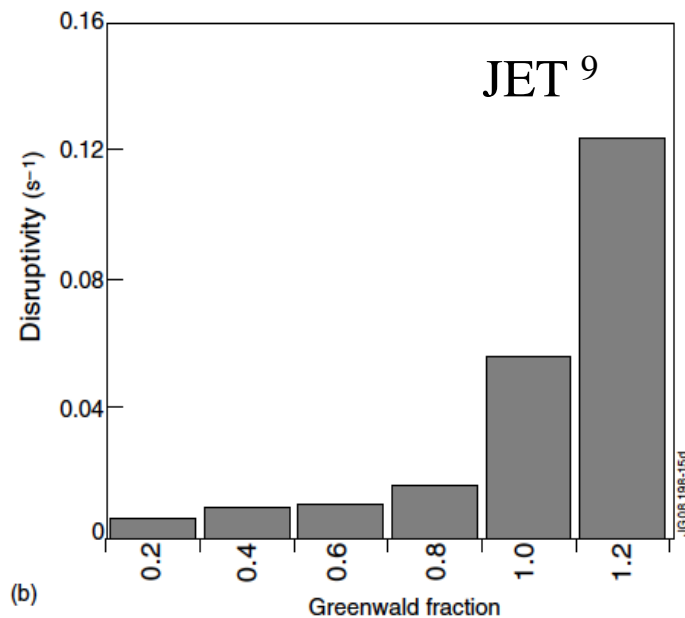
$$n_{max} \leq p_{th} R^{1.5} B^{0.5} \epsilon R^{-1.5}$$

$$n_{max} \leq \beta \epsilon B^{2.5}$$



Density: tokamak

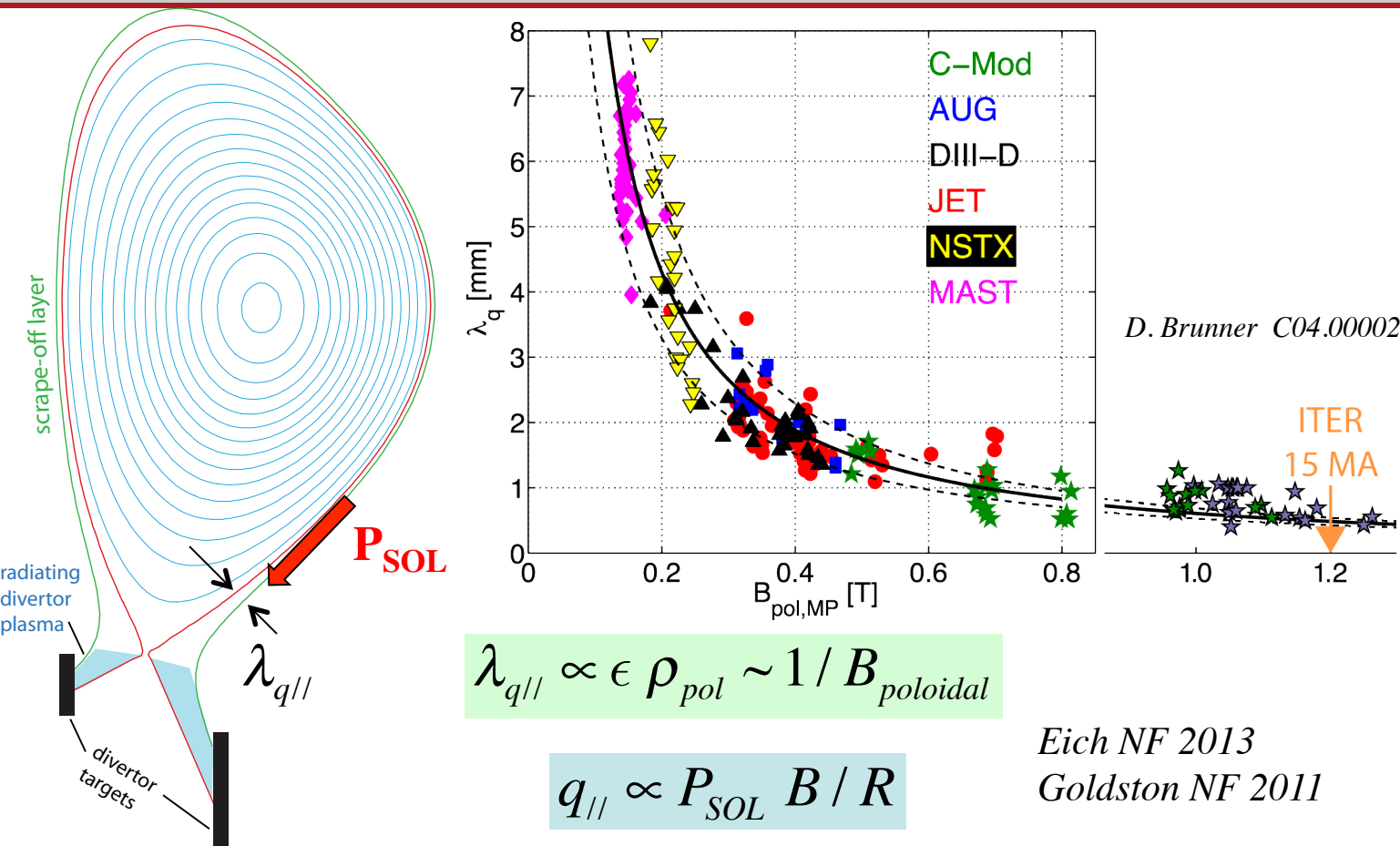
Empirical Greenwald density
is a disruptive limit in tokamaks



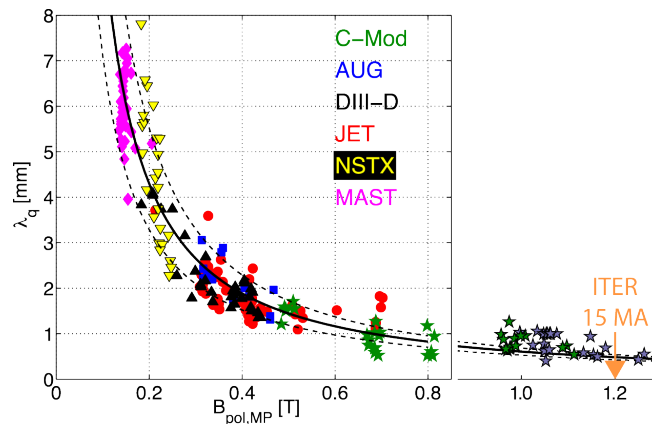
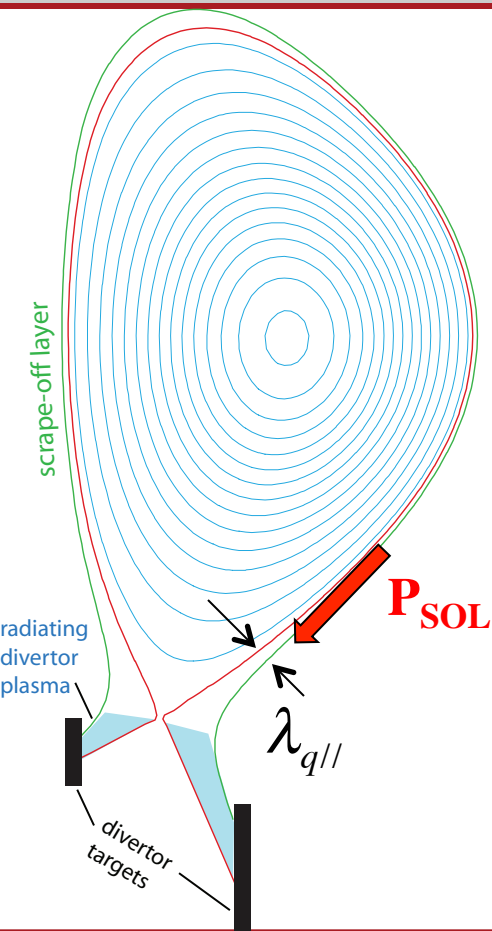
De Vries, et al. *Nucl. Fusion* **49** (2009)

$$n \leq n_{Gr} = \frac{I_p}{\pi a^2} \propto \frac{S(\kappa)}{q} \frac{B}{R}$$

Power exhaust: tokamak divertor Problem



Power exhaust: tokamak divertor Solutions



$$\lambda_{q//} \propto \epsilon \rho_{pol} \sim 1 / B_{poloidal}$$

$$q_{//} \propto P_{SOL} B / R$$

$$P_{rad} \sim n_{div}^2 f_z F(T_e)$$

$$n_{div} \sim n_{core}^2$$

$$n_{core} \propto \frac{S(\kappa)}{\epsilon} \frac{B}{R}$$

Required impurity
Fraction to Detach¹

$$c_Z \propto \frac{P_{SOL}}{B_p f_{Gr}^2}$$

Required impurity
Fraction to Dissipate
Psol in H-mode²

$$f_Z \sim B^{0.9} R^{1.3}$$

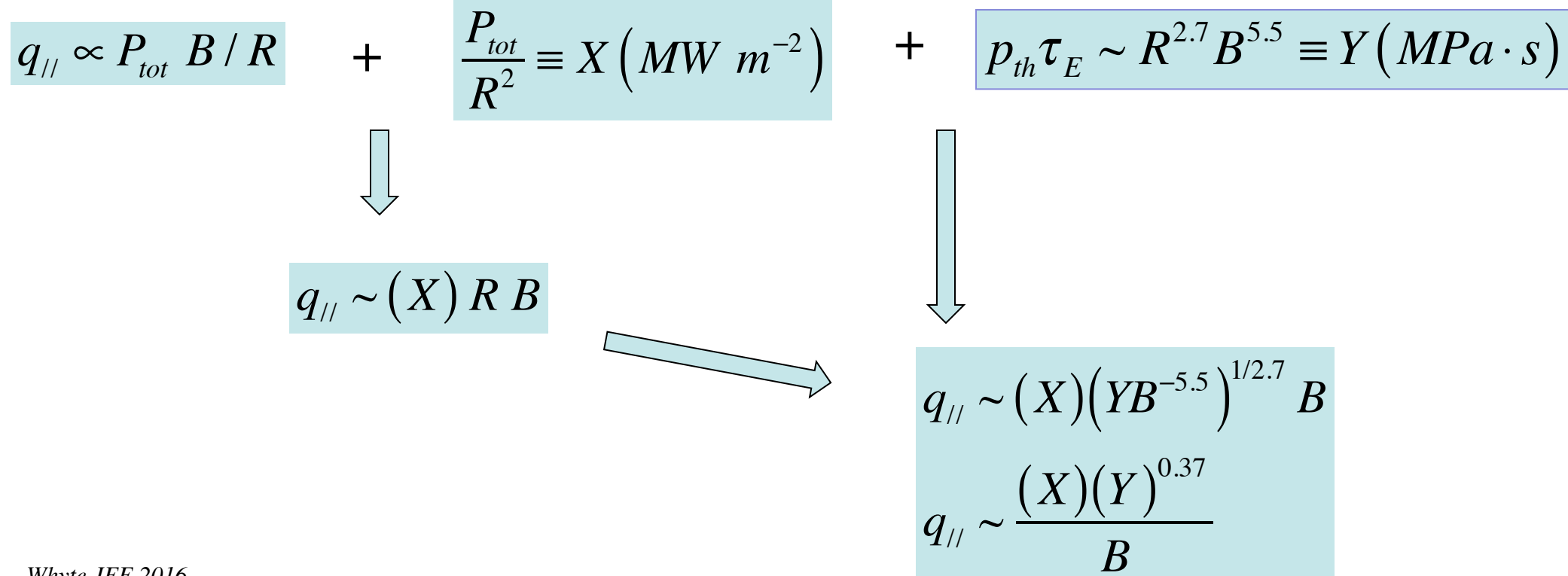
²M.L. Reinke. Nucl. Fusion 57 (2017)

¹Goldston et al PPCF 2017, APS17

Power exhaust + achieving high gain plasma

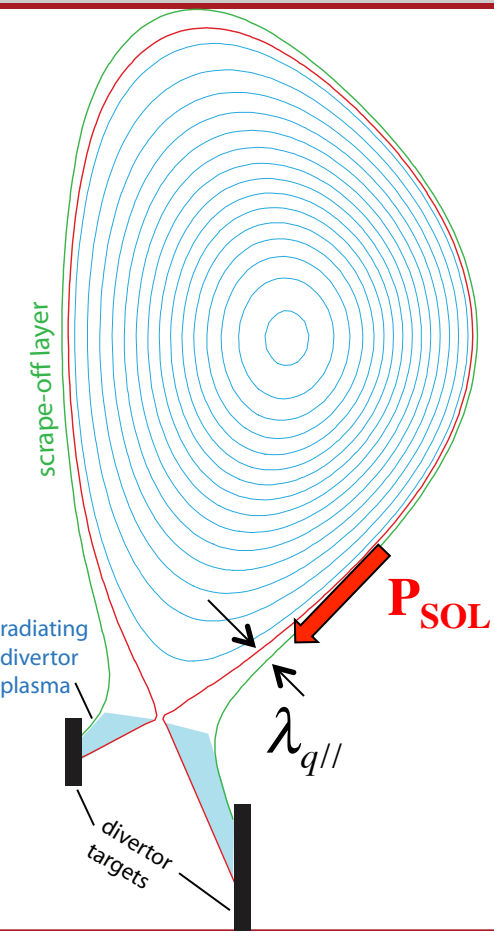
Economic target

Gain target



Whyte JFE 2016

Due to the lack of explicit size scaling in SOL heat width, high-B tokamaks have several advantages for heat exhaust



$$q_{||} \propto P_{SOL} B / R$$

$$c_Z \propto \frac{P_{SOL}}{B_p f_{Gr}^2}$$

$$f_Z \sim B^{0.9} R^{1.3}$$

$$q_{||} \sim \frac{(X)(Y)^{0.37}}{B}$$

SOL physics

Divertor detachment
(Goldston)

Impurity + H-mode
(Reinke)

Global gain +
power density

Due to the lack of explicit size scaling in SOL heat width, high-B tokamaks have several advantages for heat exhaust



Runaway e- amplification: tokamaks

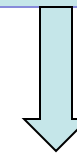
Runaway growth rate

$$\gamma_{RE} \sim \exp(I_p)$$

Rosenbluth, Putvinski Nucl. Fusion (1997)

$$p_{th} \tau_E \sim \frac{H \beta_N}{q^4} R^{2.7} B^{5.5} = X \text{ (MPa} \cdot \text{s)}$$

H₉₈
scaling



Fix aspect ratio, H, β_N

$$\frac{1}{q} \sim \frac{I_p}{BR}$$

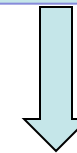
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H₉₈
scaling

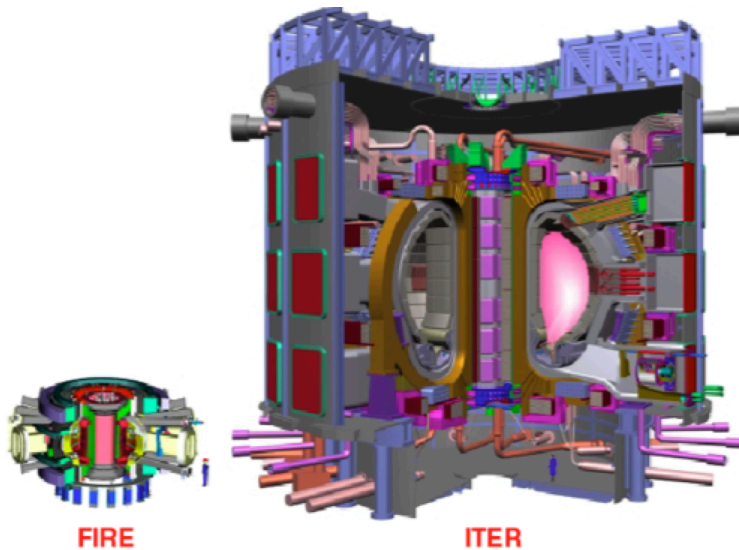


Fix aspect ratio, H, β_N

$$\frac{1}{q} \sim \frac{I_p}{BR}$$



$$I_p \sim \frac{X^{0.24} R^{0.33}}{B^{0.38}}$$



FIRE

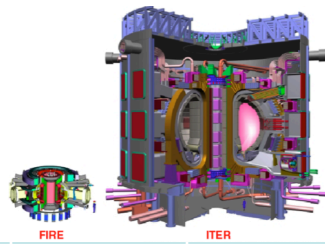
ITER

D. Meade, FED 63 (2002)

Runaway e- amplification: high-B small-R tokamaks achieve gain at lower I_p

Runaway growth rate

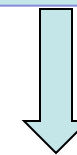
$$\gamma_{RE} \sim \exp(I_p)$$



	FIRE	ITER	ARC
B (T)	10	5.3	9.2
R (m)	2.14	6.2	3.3
Q	10	10	13
$R^{0.33}/B^{0.38}$	0.54	0.97	0.64
I_p (MA)	7.7	15	12

$$p_{th} \tau_E \sim \frac{H \beta_N}{q^4} R^{2.7} B^{5.5} = X (MPa \cdot s)$$

H_{98}
scaling



Fix aspect ratio, H , β_N

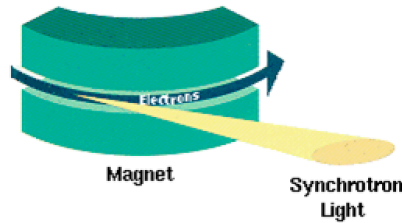
$$\frac{1}{q} \sim \frac{I_p}{BR}$$



$$I_p \sim \frac{X^{0.24} R^{0.33}}{B^{0.38}}$$

Synchrotron radiation

$$P_{\text{synchrotron}} \sim \frac{E^2 B^2}{m^4}$$



Direct loss for runaway e-

$$E_{e^-} \sim 100 \text{ MeV}$$

$$E_{\text{photon}} \sim \text{eV, visible}$$

Following Rosenbluth's treatment¹

$$P_{\text{synch}} \approx 8 \times 10^{-7} \frac{n_{20} T_{\text{keV}}^{2.75} B^{1.5}}{(\beta_e R_m)^{1/2}}$$

¹Rosenbluth Nucl. Fusion **10** (1970)

*M. Bakhtiari et al Phys Rev Lett **94** (2005)*

A. Tinguely APS 2017

Thermal electrons

Photons $\sim$ mm-m wavelength

Synchrotron radiation is a soft limit to $B_0 > 15$ T

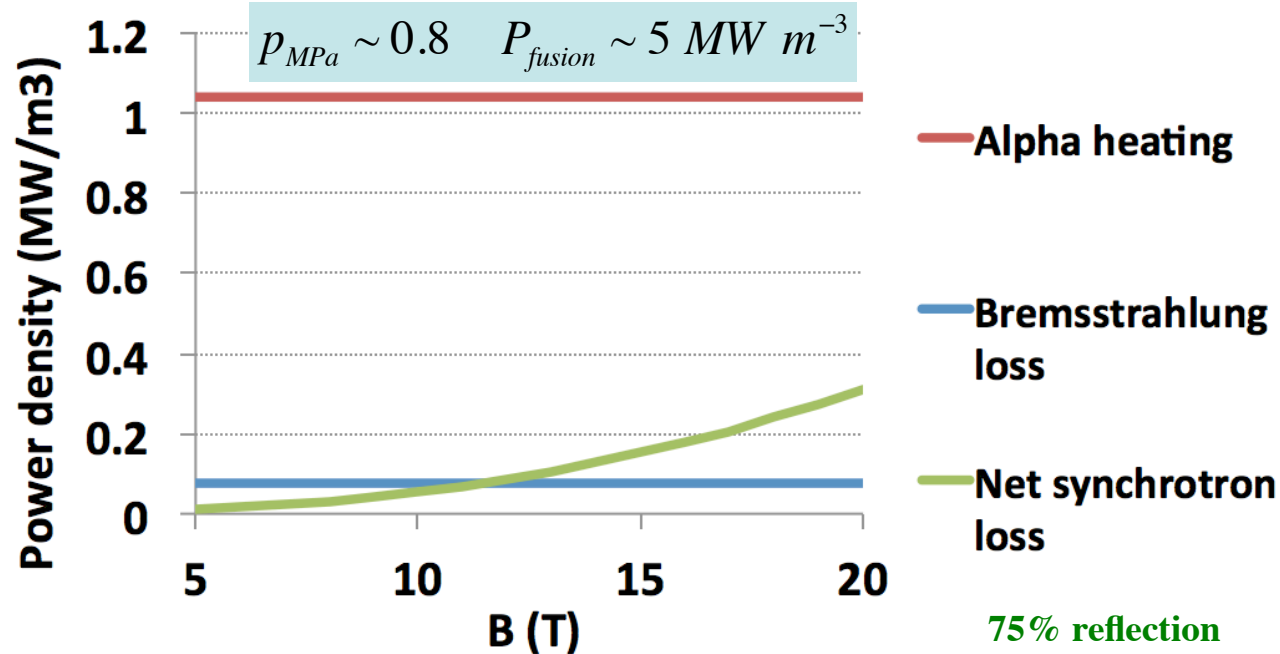
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$$P_{\text{brem}} \sim 0.005 n_{20}^2 T_{\text{keV}}^{1/2} [MW m^{-3}]$$

$$n_{20} = 2.1 \quad T_{\text{keV}} = 12 \quad R_m = 5$$



High-B access to higher density, lower T solutions mitigates synchrotron losses (and TAEs)

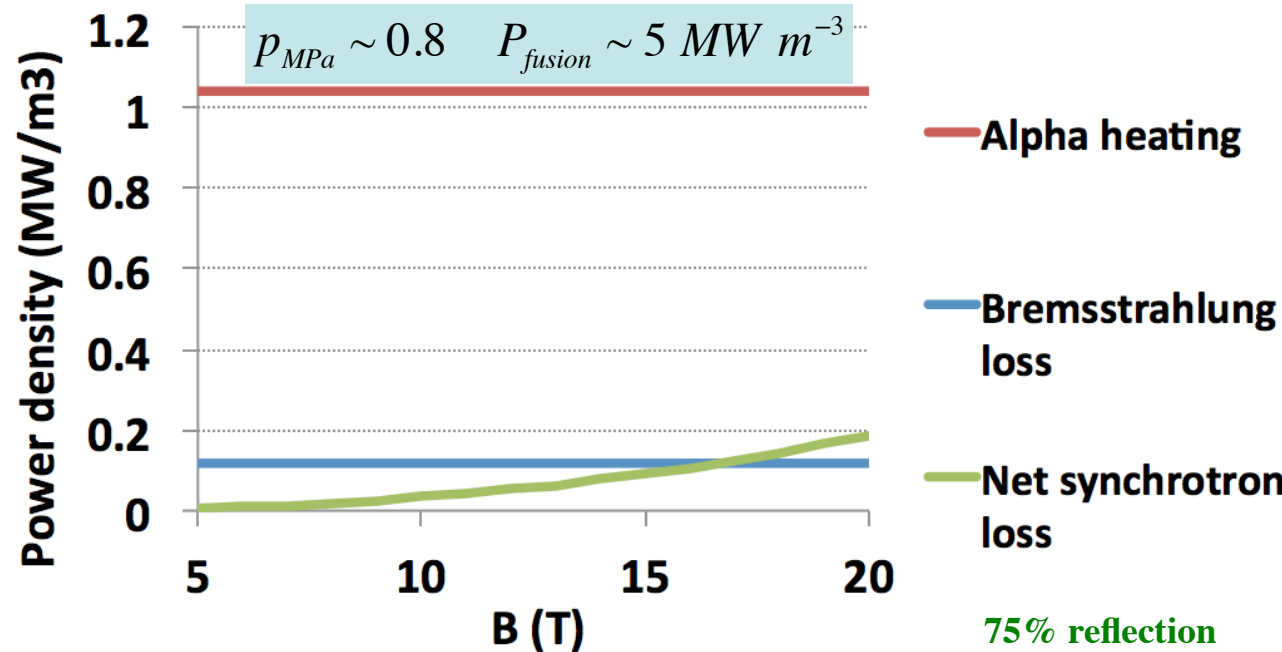
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$$P_{\text{brem}} \sim 0.005 n_{20}^2 T_{\text{keV}}^{1/2} [MW m^{-3}]$$

$$n_{20} = 2.8 \quad T_{\text{keV}} = 9 \quad R_m = 5$$



Am I happy or sad?

Issue	Scaling	
Power density	B^4	1
Confinement (generic)	$R^2 B^2$	1
Confinement (tokamak)	$R^{2.7} B^{3.5}$ (H ₉₈) $R^{3.1} B^{2.1}$ (Petty)	1
Confinement (stellarator)	$R^{2.8} B^{2.1}$	1
Gain	$R^{2-3.1} B^{4-5.5}$	1
Stable pedestal/I-mode	$\sim \beta_N B^2$	1

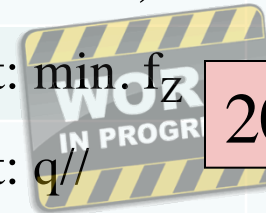
Issue	Scaling	
Density (tokamak)	$R^{-1} B^1$	1
Density (stellarator)	$\beta B^{2.5}$ (burning)	1
Heat exhaust: min. f_z	$R^{1.3} B^{0.9}$	1
Heat exhaust: $q//$	B^{-1} (burning)	1
Runaway e- amp.	$\exp(R^{0.28} / B^{0.3})$	1
Synchrotron: runaways	B^2	1
Synchrotron: thermal	$\sim B^{1.5}$	1
TAE	$n \sim B, v_A \sim B$	1



Am I happy or sad? *I'm happier than in 2002*

Issue	Scaling	
Power density	B^4	①
Confinement (generic)	$R^2 B^2$	①
Confinement (tokamak)	$R^{2.7} B^{2.1}$ (Petty)	①
	1998	
	2008	
Confinement (stellarator)	$R^{2.8} B^{2.1}$	①
	2005	
Gain	$R^{2-3.1} B^{4-5.5}$	①
Stable pedest	$\beta_N B^2$	①
	2010	
	2016	

Issue	Scaling	
Density (tokamak)	$R^{-1} B^1$	①
Density (stellarator)	$\beta B^{2.5}$ (burning)	①
Heat exhaust: min. f_z	$R^{1.2} B^{0.9}$	①
Heat exhaust: $q//$	B (burning)	①
Runaway e- amp.	$\exp(R^{0.28} / B^{0.3})$	①
Synchrotron: runaways	B^2	①
Synchrotron: thermal	$\sim B^{1.5}$	①
TAE	$n \sim B, v_A \sim B$	①



2010-17

2005-17

Some of the high-field pioneers



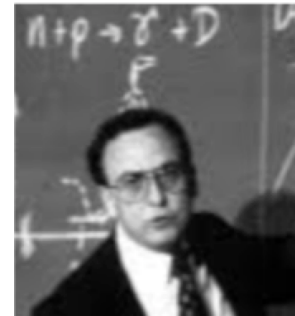
Bruno
Coppi



Bruce
Montgomery

US Patent 1973

**Device for plasma confinement and heating by
high currents and non-classical plasma
transport properties**
US 3778343 A

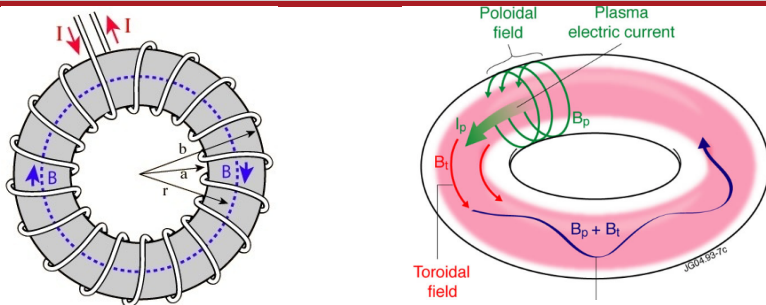


Ron Parker



Dale Meade

Electromagnets & Tokamak plasmas: same physics



$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

Ampere's law

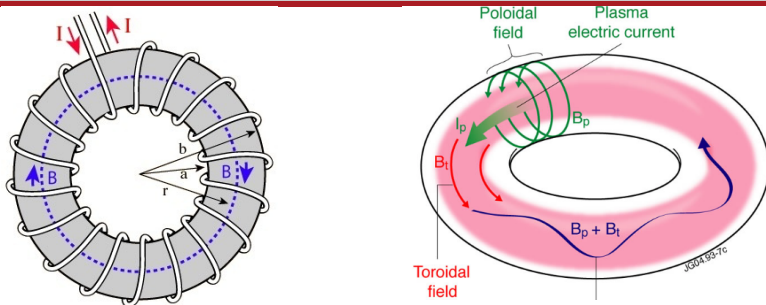
$$\nabla p = \vec{j} \times \vec{B}$$

Force balance

$$P = \eta j^2$$

Ohmic heating

Electromagnets & Tokamak plasmas: same physics



$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

Ampere's law

$$B \sim j$$

$$\nabla p = \vec{j} \times \vec{B}$$

Force balance

$$P_{\text{magnet}} \sim B^2$$

$$P = \eta j^2$$

Ohmic heating

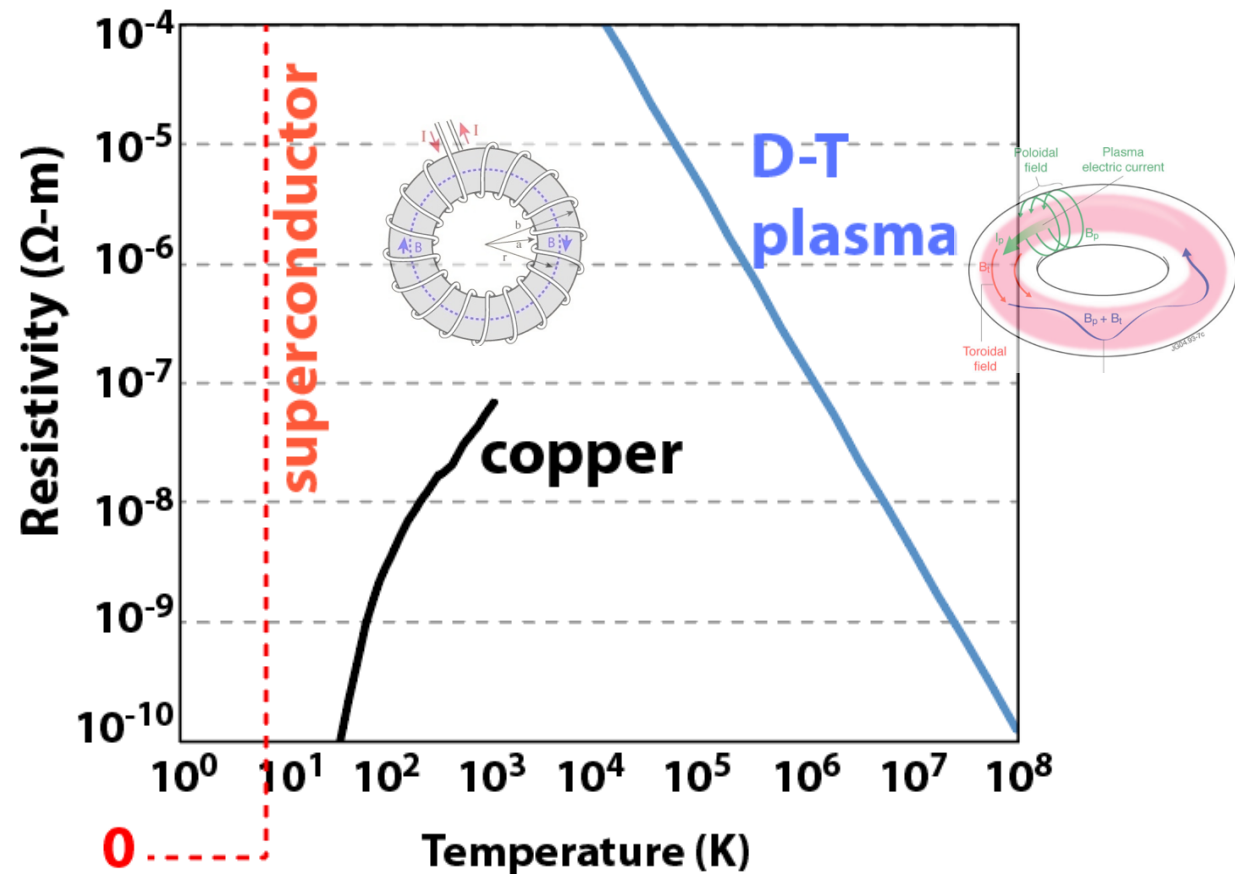
$$P_{\text{magnet}} \sim B^2$$

Same physics: different media

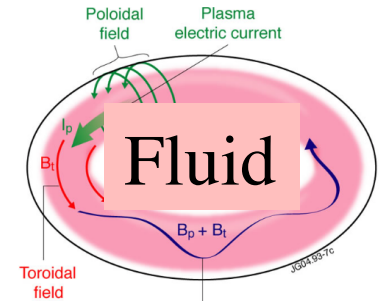
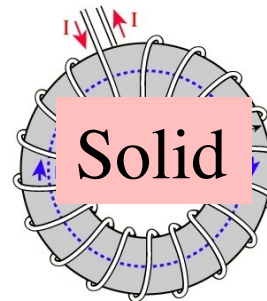
$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla p = \vec{j} \times \vec{B}$$

$$P = \eta j^2$$



Same physics: different media



$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla p = \vec{j} \times \vec{B}$$

$$P = \eta j^2$$

Solid: “resistant to changes in shape or volume”

Fluid: “a substance which cannot resist deformation under shear stress applied to it”

Analytic assessment of simple cylindrical solenoid magnet coil provides insight to limit in B

$j \times B$ force

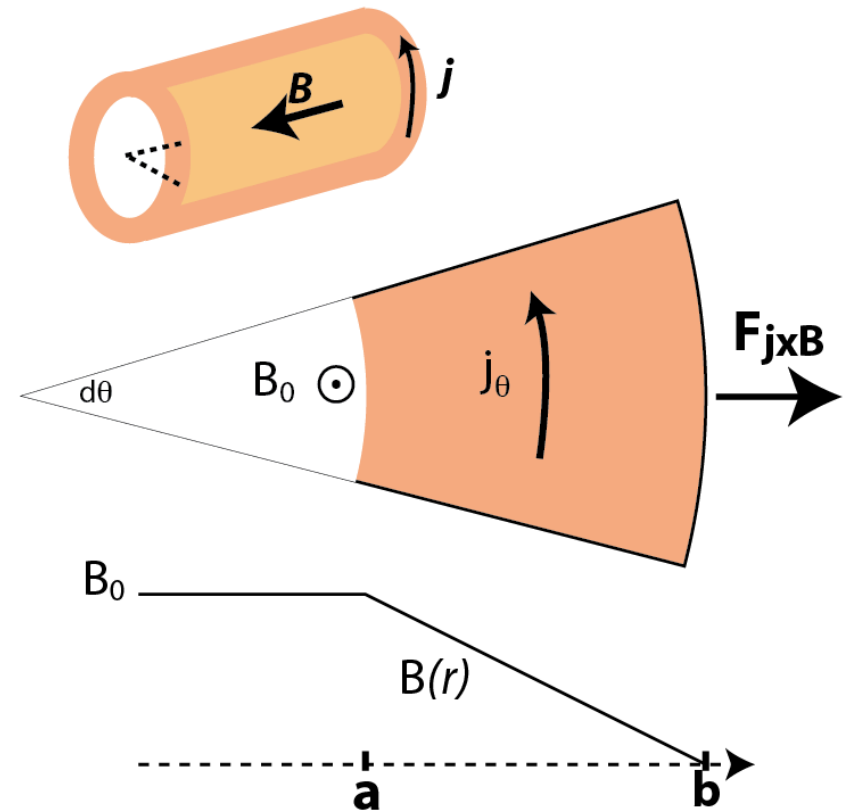
$$F_{j \times B} = \int_a^b j_\theta B_z r d\theta dr dz$$

Ampere's law /w uniform j

$$B_z(r) = \mu_0 j (b - r)$$

Outward force per segment

$$\begin{aligned} \frac{F_{j \times B}}{d\theta dz} &= j^2 \int_a^b (b - r) r dr \\ &= \frac{B^2}{2\mu_0} \frac{2}{(b - a)^2} \int_a^b (b - r) r dr \end{aligned}$$



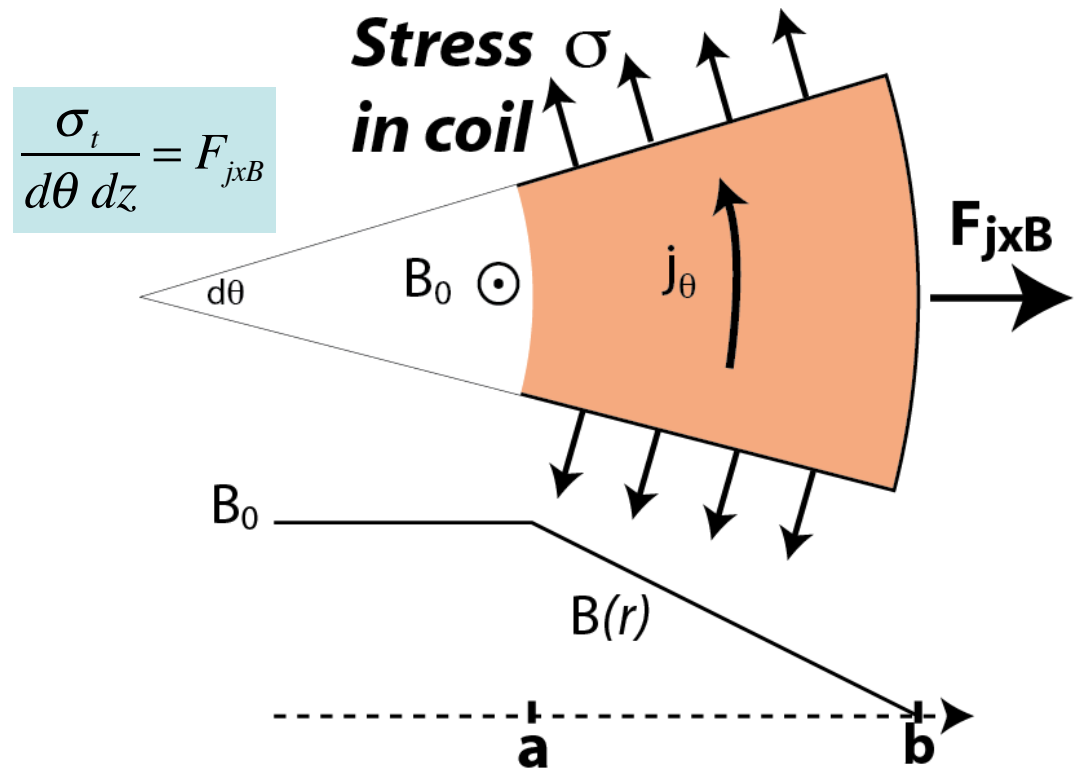
jxB forces must be taken by the stress in the coil, which has intrinsic limits

Tensile stress must balance jxB force

$$\int_a^b \sigma_t \sin \theta \, d\theta \, dr \, dz = F_{j \times B}$$

$$\sigma_t = \frac{B^2}{2\mu_0} \frac{\int_a^b (b-r)r \, dr}{(b-a)^3}$$

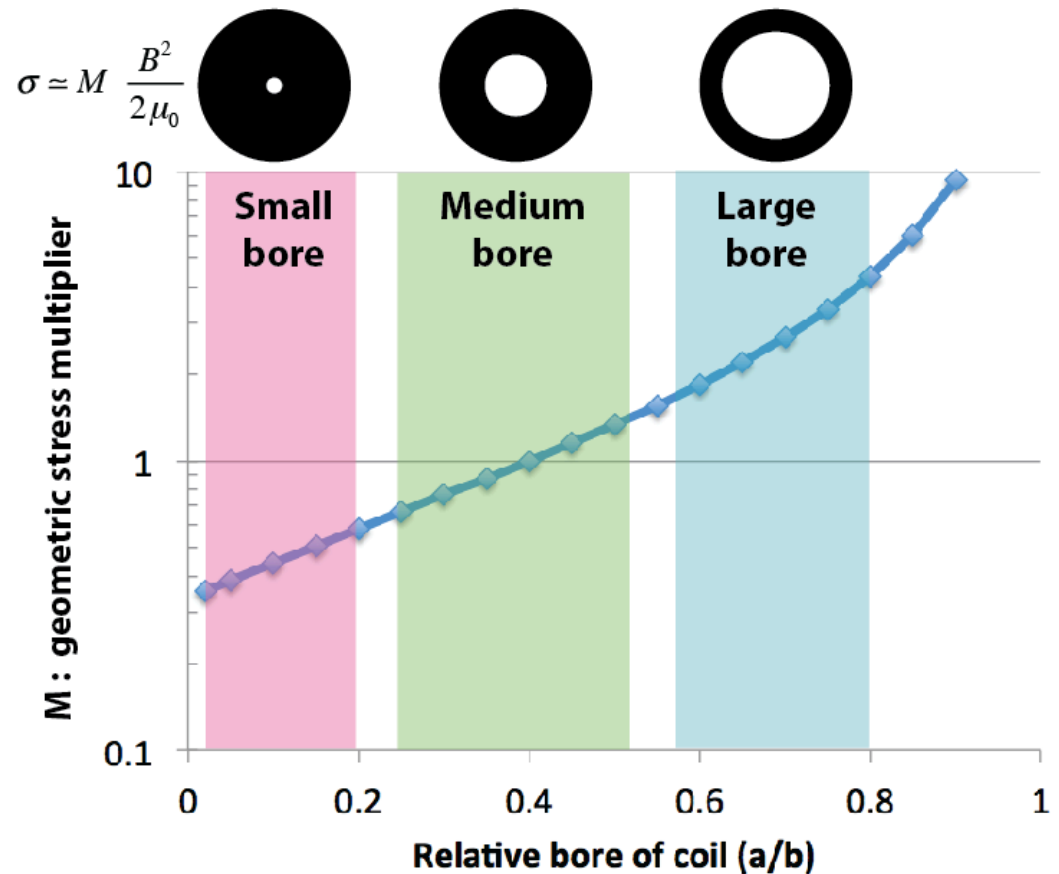
$$\sigma_t = \frac{B^2}{2\mu_0} \left[\frac{2a+b}{3(b-a)} \right]$$



As in toroidal plasma physics, aspect ratio is a critical and complex optimization

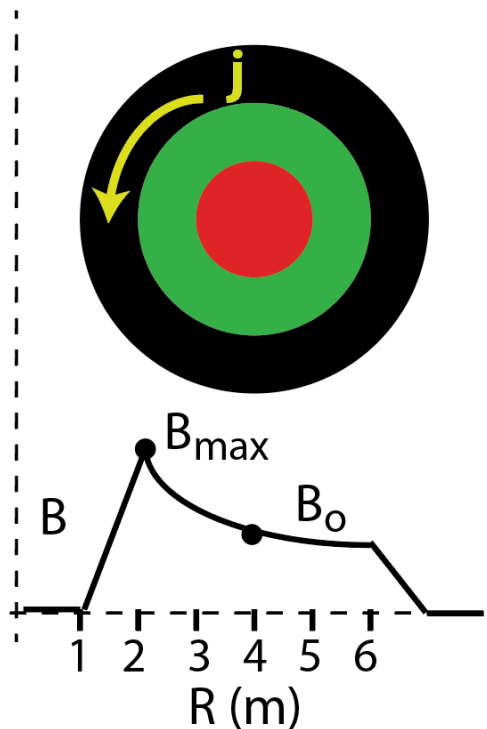
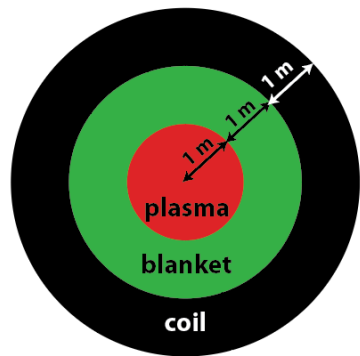
$$x \equiv \frac{a}{b}$$

$$M = \frac{2x+1}{3(1-x)}$$



Simple toroidal “solenoid” to explore limits

$R=4$ m, $A=4$, $B_0=B_{\max}/2$

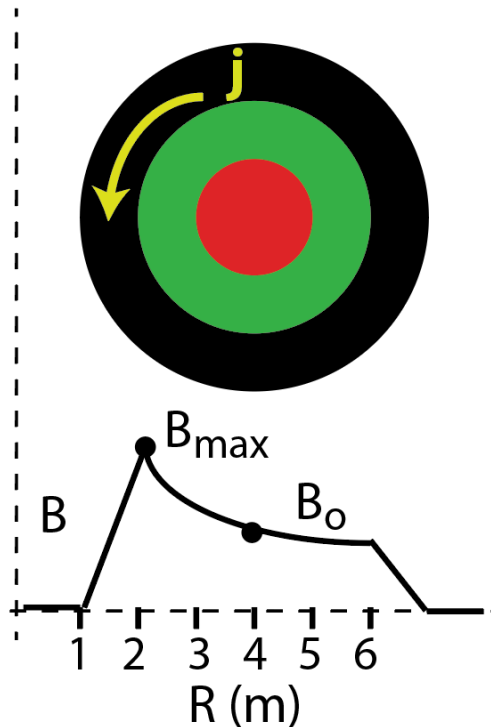
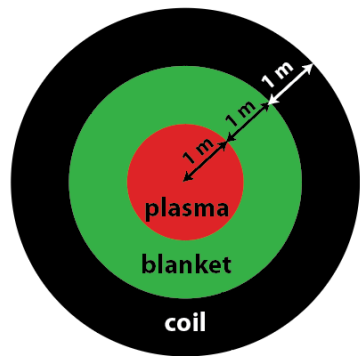


$$B = \frac{\mu_0 \int j_z \pi R dR}{2\pi R}$$

$$B_{\max} = 0.3\pi j_{MA/m^2}$$

Simple toroidal “solenoid” to explore limits

$R=4$ m, $A=4$, $B_0=B_{\max}/2$



$$B = \frac{\mu_0 \int j_z \pi R dR}{2\pi R}$$

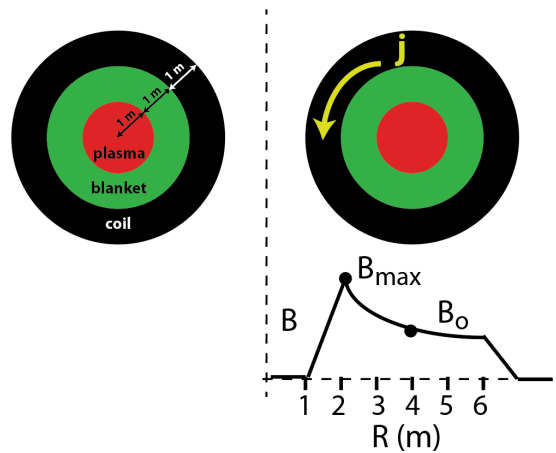
$$B_{\max} = 0.3\pi j_{MA/m^2}$$

$$x \equiv \frac{a}{b} = \frac{2}{3}$$

$$M = 2.3$$

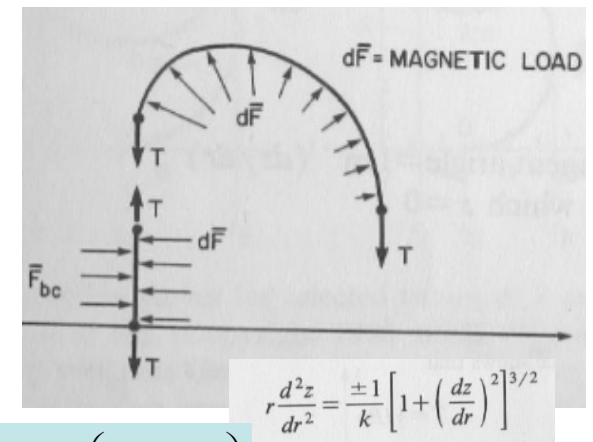
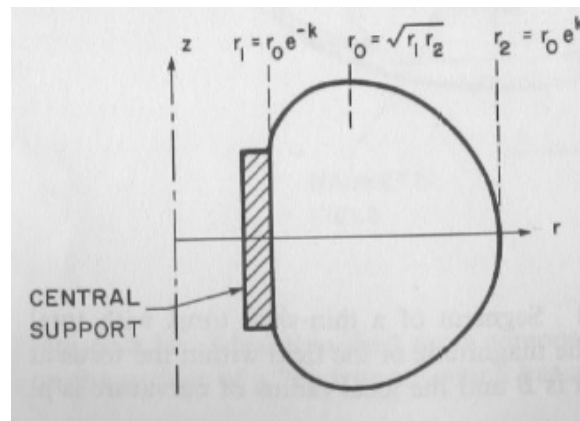
$$\sigma_{\max} [MPa] \simeq M \frac{B_{\max}^2}{2\mu_o}$$

D-shaped coils are actually used for shaped plasmas..but stress limits ~ equal for test case



$$\sigma_{\max} \approx 2.3 \left(\frac{B^2}{2\mu_0} \right)$$

Constant tension coil



$$\sigma_{\text{column}} = \left(\frac{B^2}{2\mu_0} \right) \frac{2\pi R_{\text{coil},\min}^2}{A_{\text{column}}} \ln \left(\frac{R_{\text{coil},\max}}{R_{\text{coil},\min}} \right)$$

$$\sigma_{\max} \approx 2.2 \left(\frac{B^2}{2\mu_0} \right)$$

Thome & Tarrh
"MHD and Fusion Magnets"
John Wiley & Sons, NY, 1982

Explore magnet concepts

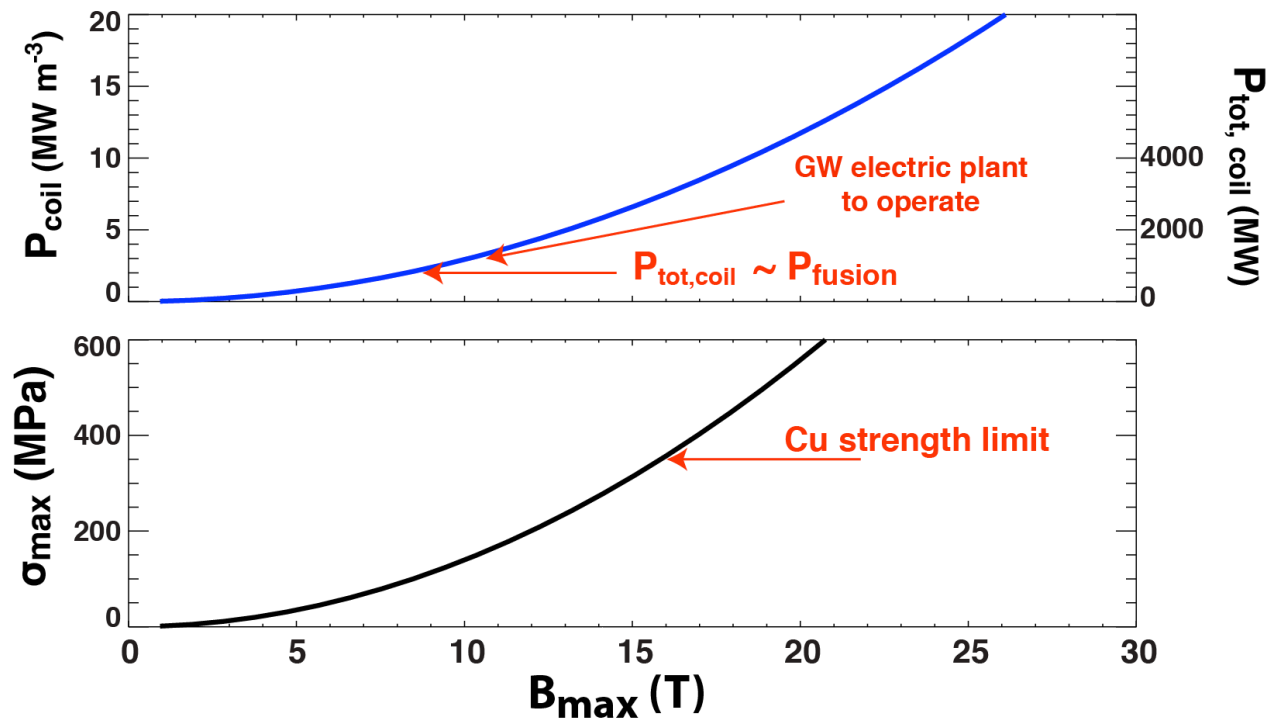
Concept	$B_{\max}$ (T)	T (K)
Water-cooled Cu		
Cryogenic Cu		
Nb-Ti Superconductor		
Nb-Sn Superconductor		
REBCO Superconductor		

Room-temperature water-cooled Cu coils

Coil cross-section



2/3 Cu
1/3 H₂O



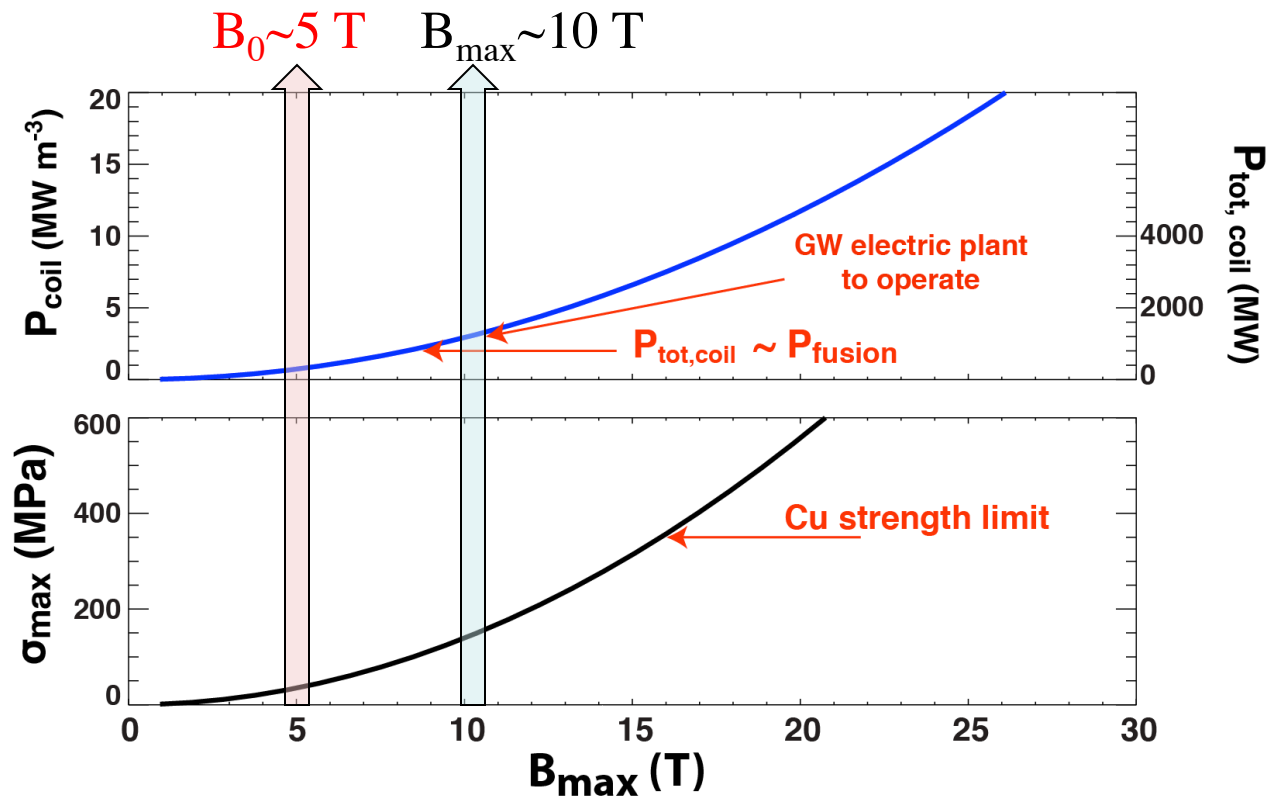
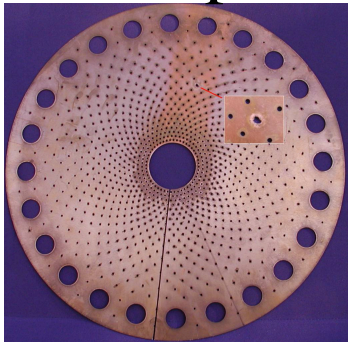
Room-temperature water-cooled Cu coils

Coil cross-section

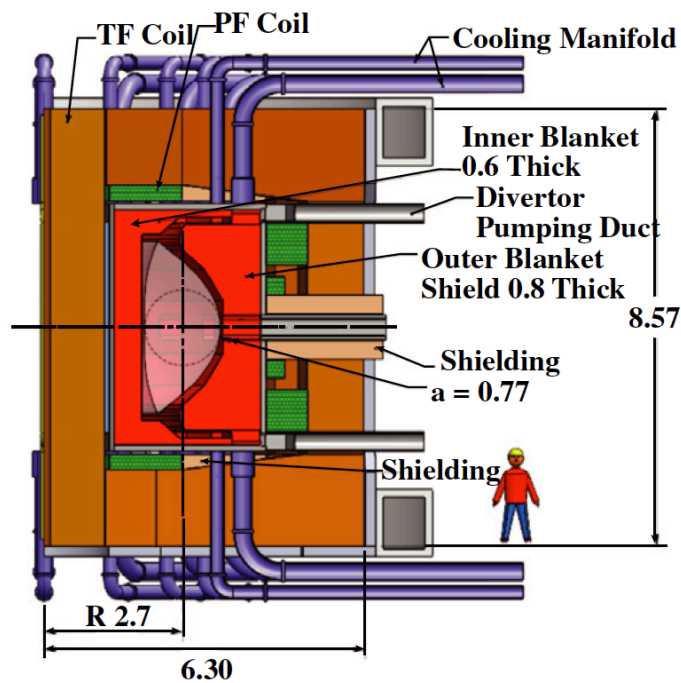


2/3 Cu
1/3 H₂O

Bitter plate



Room-temperature water-cooled Cu coils



FDF¹

$B_{\max} \sim 12$ T

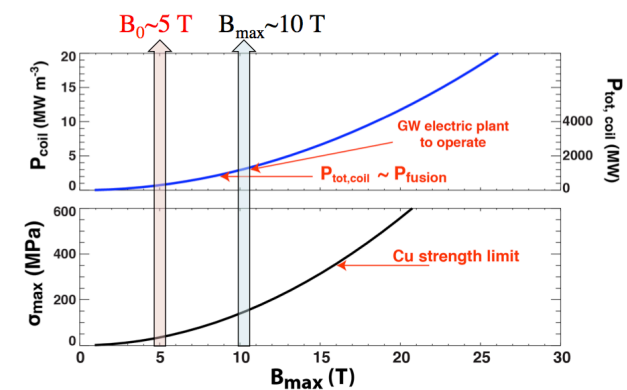
$B_0 \sim 5.4$ T

$R \sim 2.7$ m

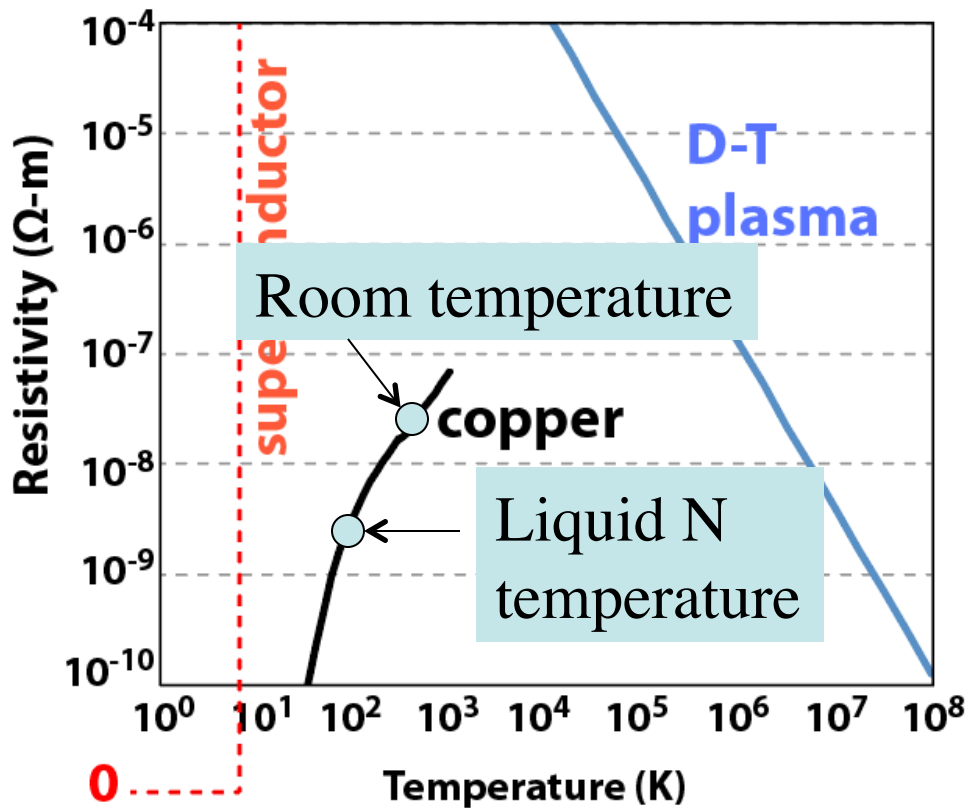
$P_{\text{tot,coil}} \sim 500$ MW

$P_f \sim 290$ MW

¹Chan et al Nucl Fusion 51 (2011)



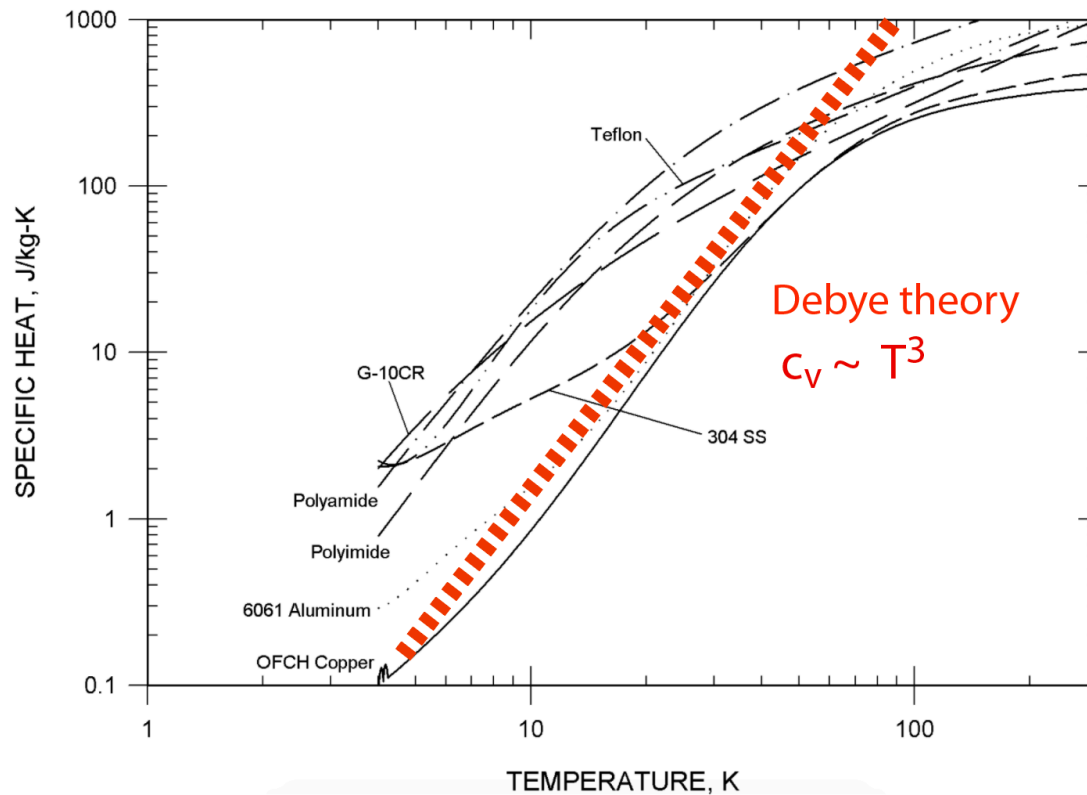
Material properties, resistivity and heat capacity, are highly T dependent at cryogenic temperatures



Cyrogenic material properties

	Cu	SS (304 L)
Tensile strength (MPa)	200	1340
Resistivity (nohm-m)	~2	500

Cryogenic properties are highly sensitive to T



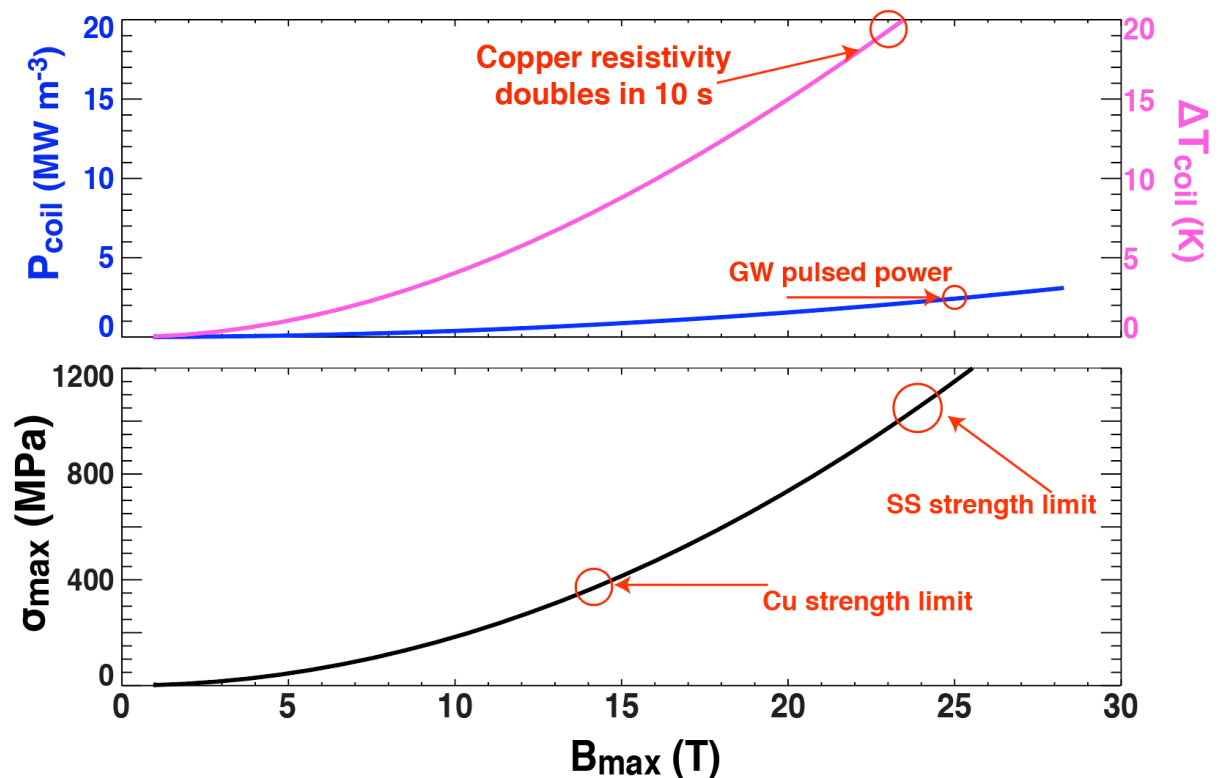
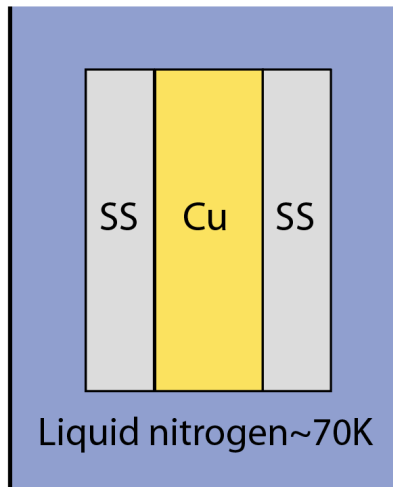
$$\frac{dT}{dt} = \frac{\eta j^2}{\rho c_v} \propto \frac{1}{T}$$



$$T_f = \left[T_o^2 + 15(\eta j^2)_{MWm^{-2}} \cdot \Delta t \right]^{1/2}$$

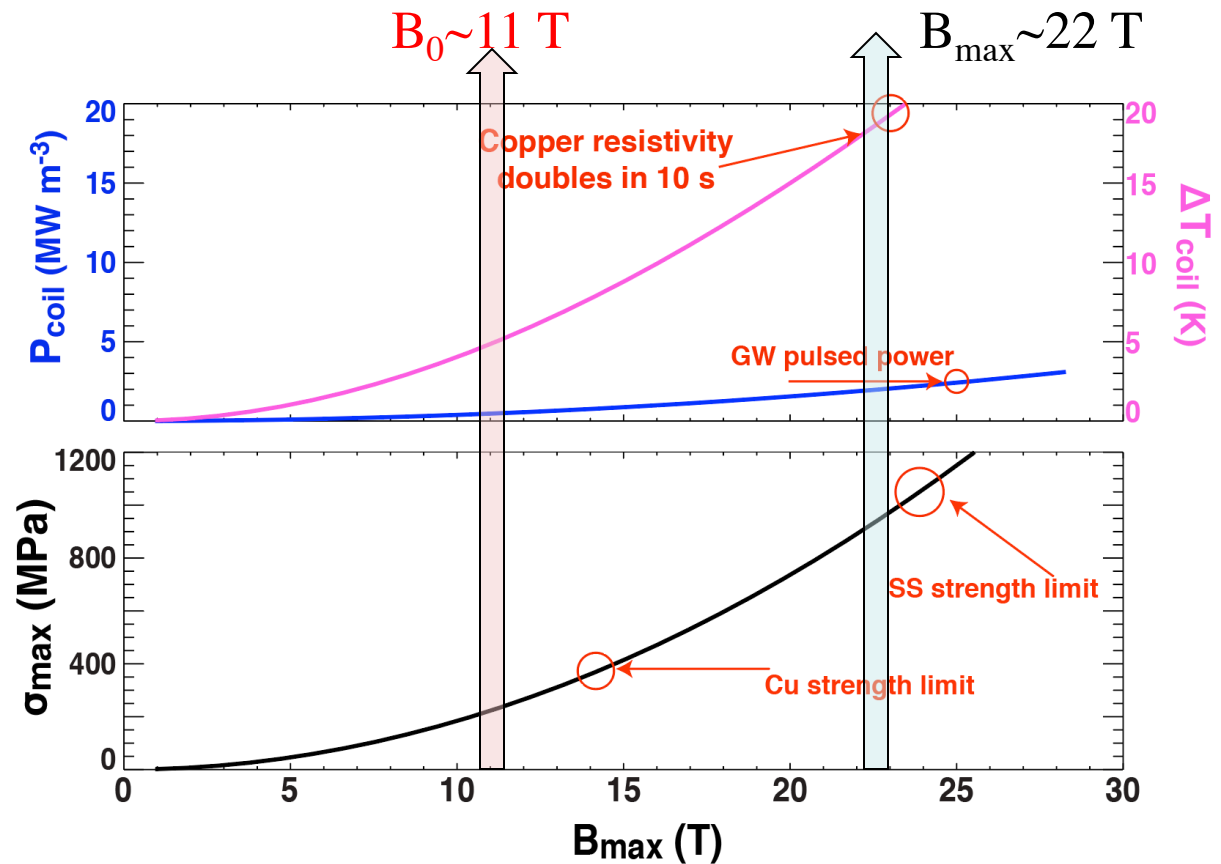
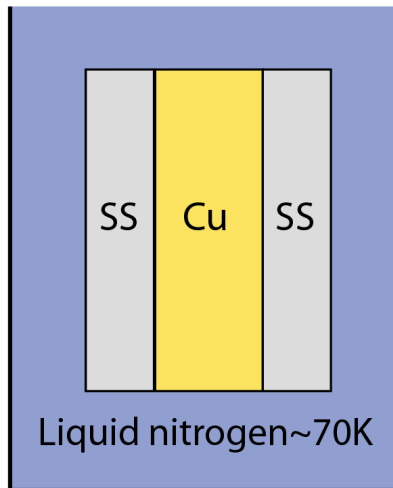
LN-cooled copper + steel for stress loading

Pulsed due to lack of active cooling

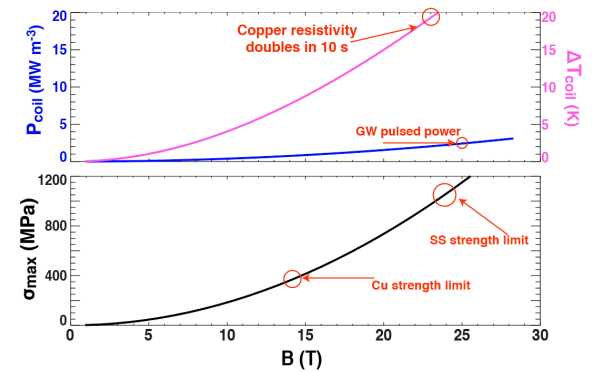
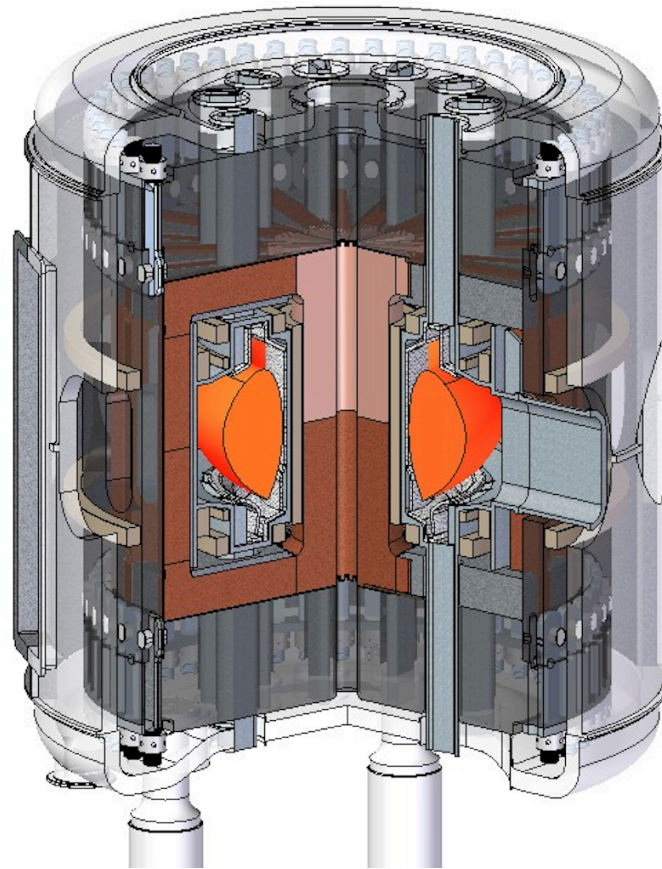
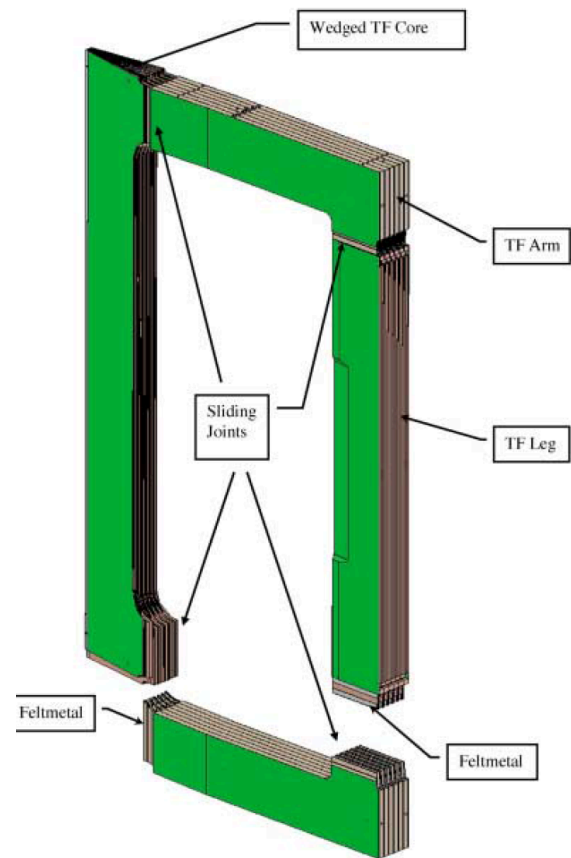


LN-cooled copper + steel for stress loading

Pulsed due to lack of active cooling



The “Alcator” approach enforces Cu plates with SS + translates forces to steel plates + pressure vessel

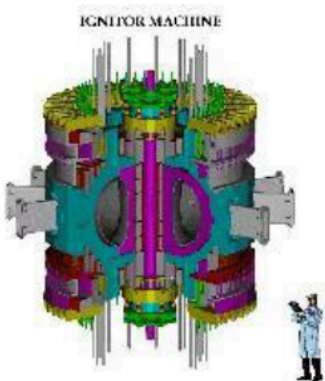


J. Irby et al FST 51 (2007)

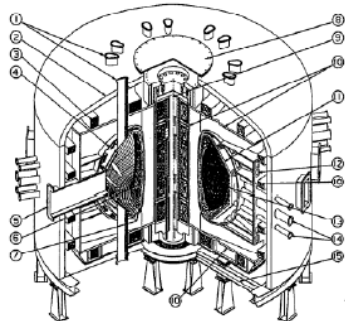
Whyte, Case for High Field Fusion, APS 2017

Cryogenic-Cu magnets access $B_{\text{coil}} > 20\text{T}$, $B_0 \sim 10\text{ T} \rightarrow$ pulsed high-gain, burning plasmas at very modest size

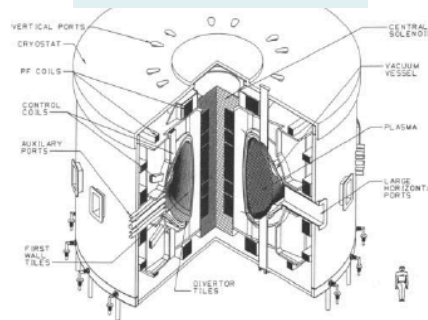
Ignitor



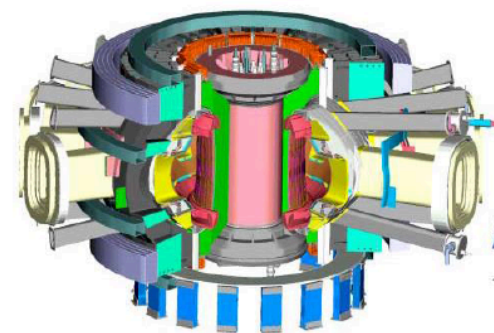
Compact Ignition Tokamak



Burning Plasma eXperiment



Fusion Ignition Research Exp't



B: 13 T

10.4 T

9 T

10 T

R: 1.3 m

1.2 m

2.6 m

2.14 m

B. Coppi et al Nucl Fusion **41** (2001) Schmidt, Furth PPPL 1986

Goldston FST **21** (1992)

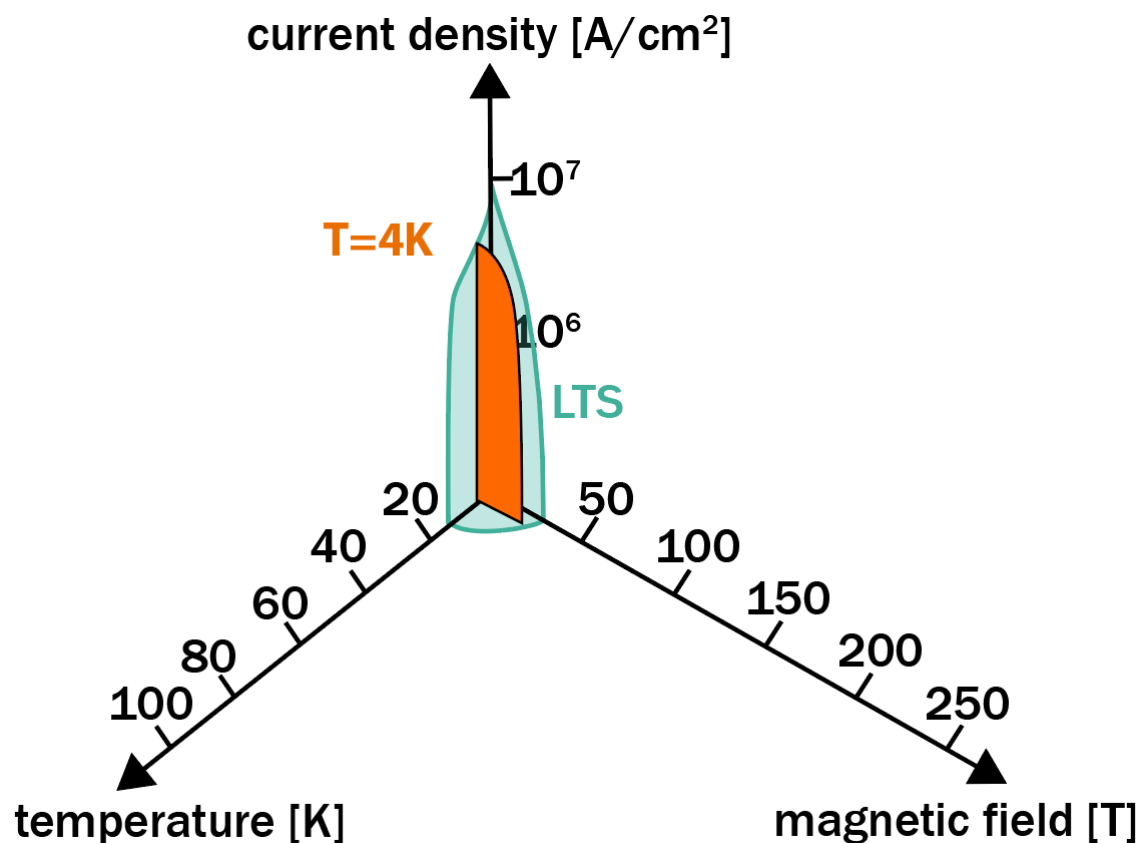
D. Meade, FED **63** (2002)

Whyte, Case for High Field Fusion, APS 2017

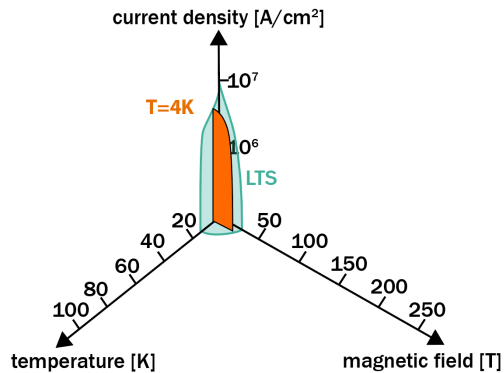


PSFC

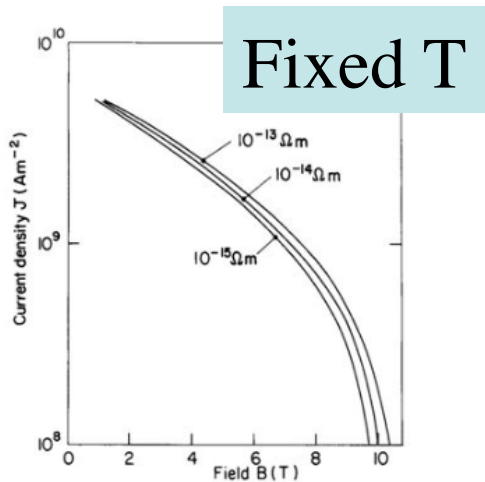
Superconductors: zero resistivity, but a restricted operating space in T, j and B



Superconductors: critical current, at fixed T, depends on SC type and B



$$\frac{J_c}{J_{c,0}} = \left(\frac{B}{B_0} \right)^{-\alpha}$$

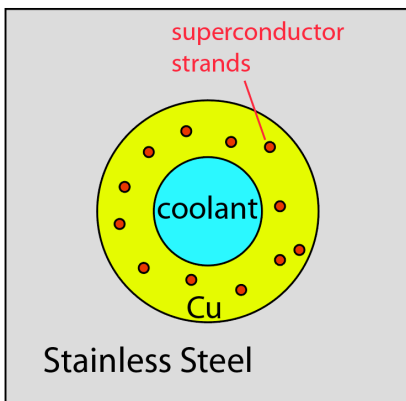


	J_{c0}	B_0	α
Nb-Ti	10^3	5	3
Nb ₃ -Sn	10^3	10	3

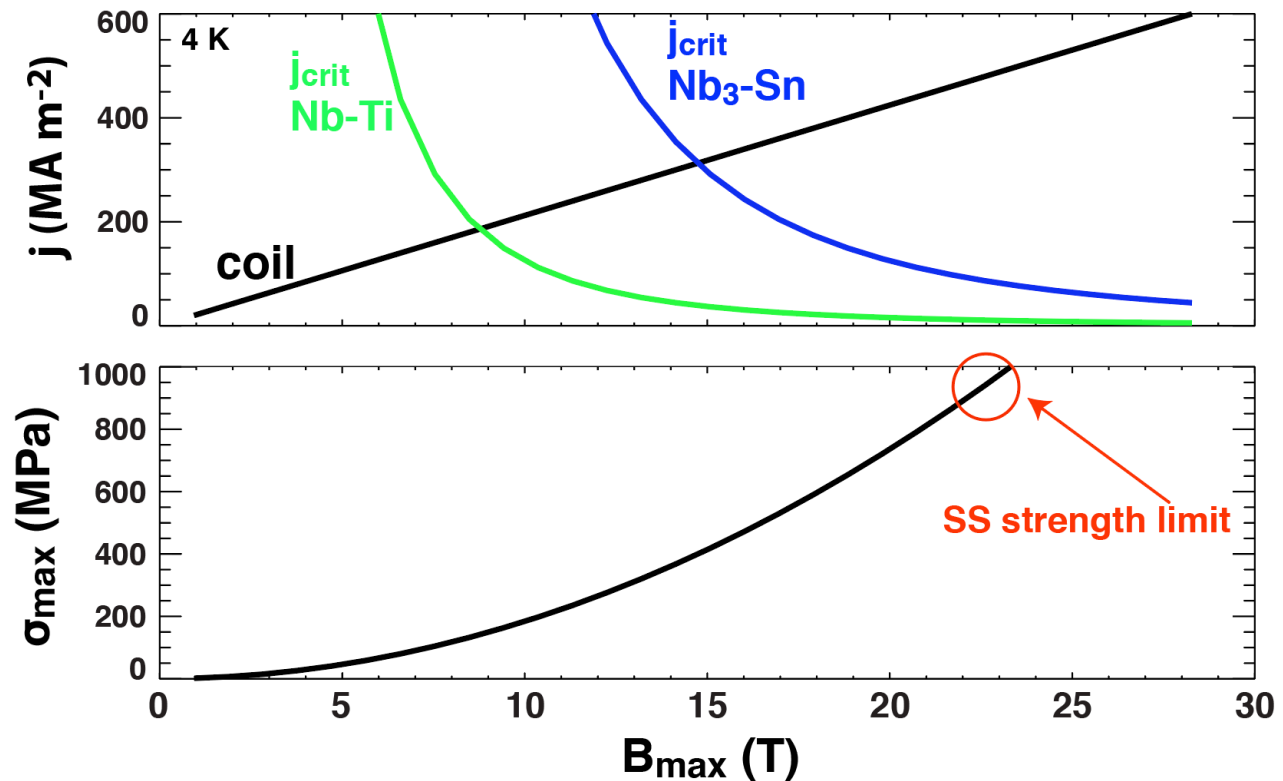
$$T \sim 4 \text{ K}, B > B_0$$

Nb-based superconductors: B limited by critical current at $T \sim 4\text{K}$

Coil Cross-section

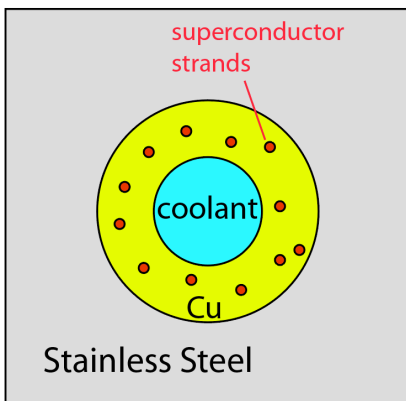


50% SS
22% cool
25% Cu
3 % SC

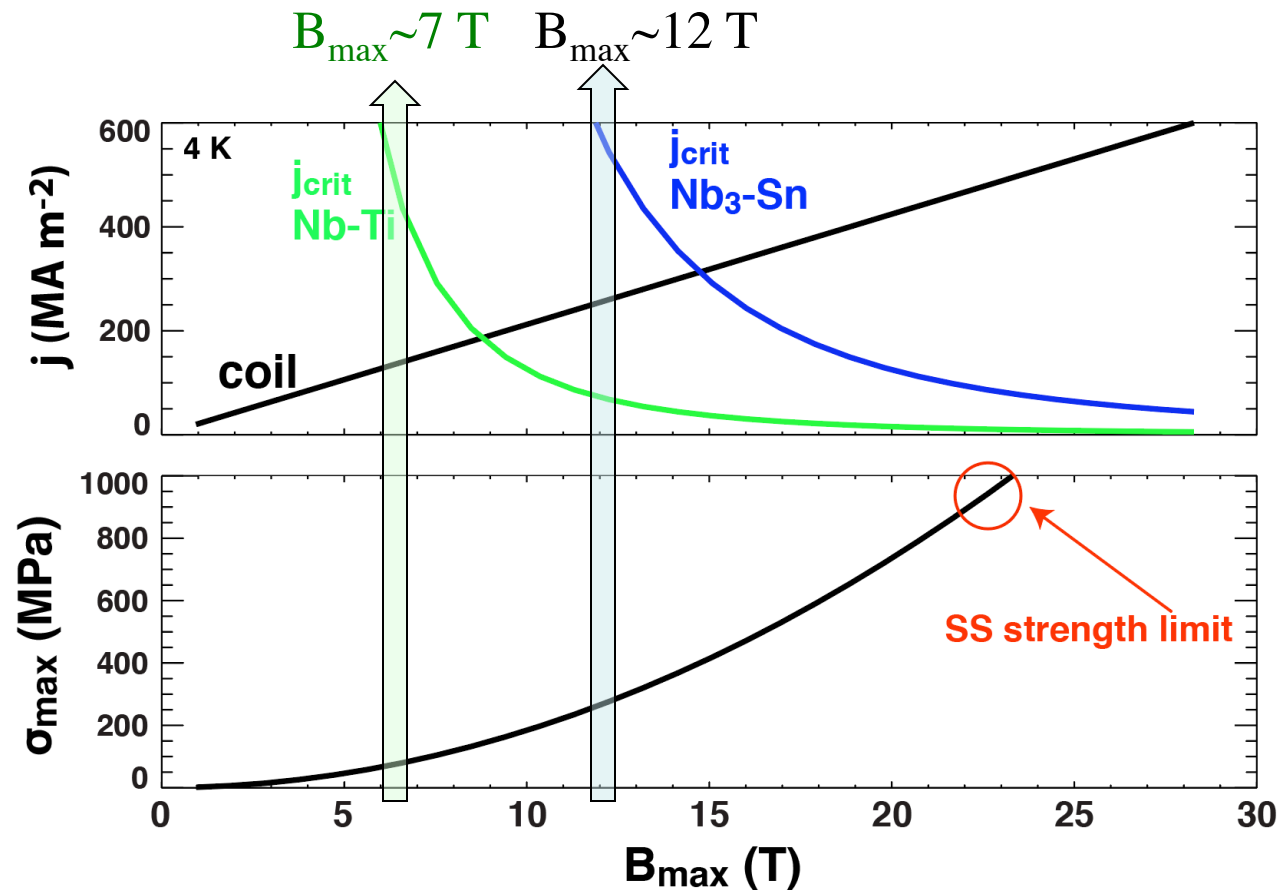


Nb-Sn superconductors: B limited by critical current at $T \sim 4\text{K}$

Coil Cross-section

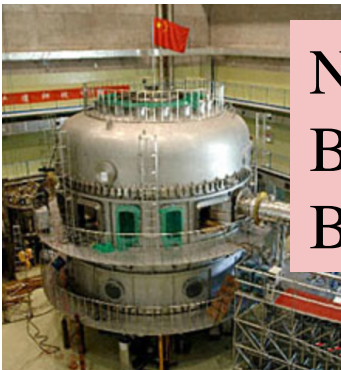


50% SS
22% cool
25% Cu
3 % SC



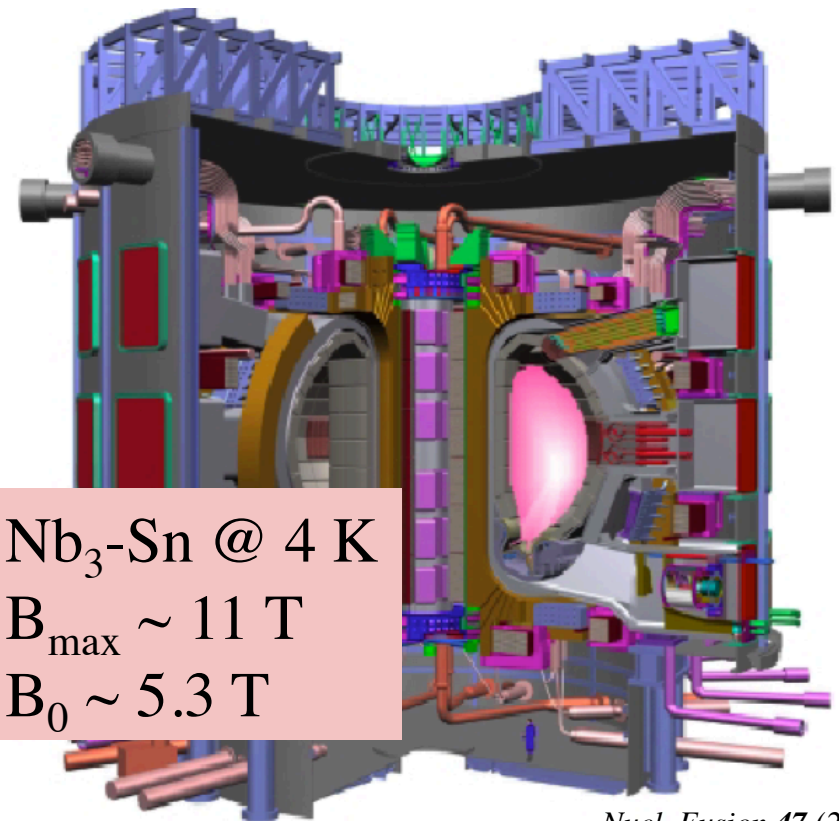
Superconductor tokamak examples: EAST & ITER

EAST



Nb-Ti @ 4 K
 $B_{\max} \sim 7 \text{ T}$
 $B_0 \sim 3.5 \text{ T}$

Yuanxi et al Plasma Sci Tech (2006)



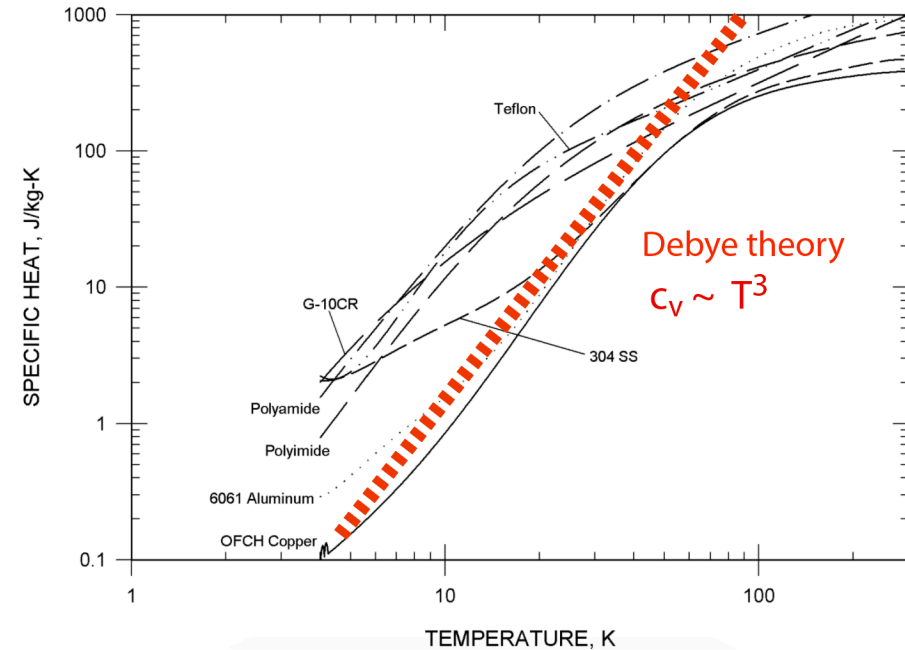
$\text{Nb}_3\text{-Sn}$ @ 4 K
 $B_{\max} \sim 11 \text{ T}$
 $B_0 \sim 5.3 \text{ T}$

ITER

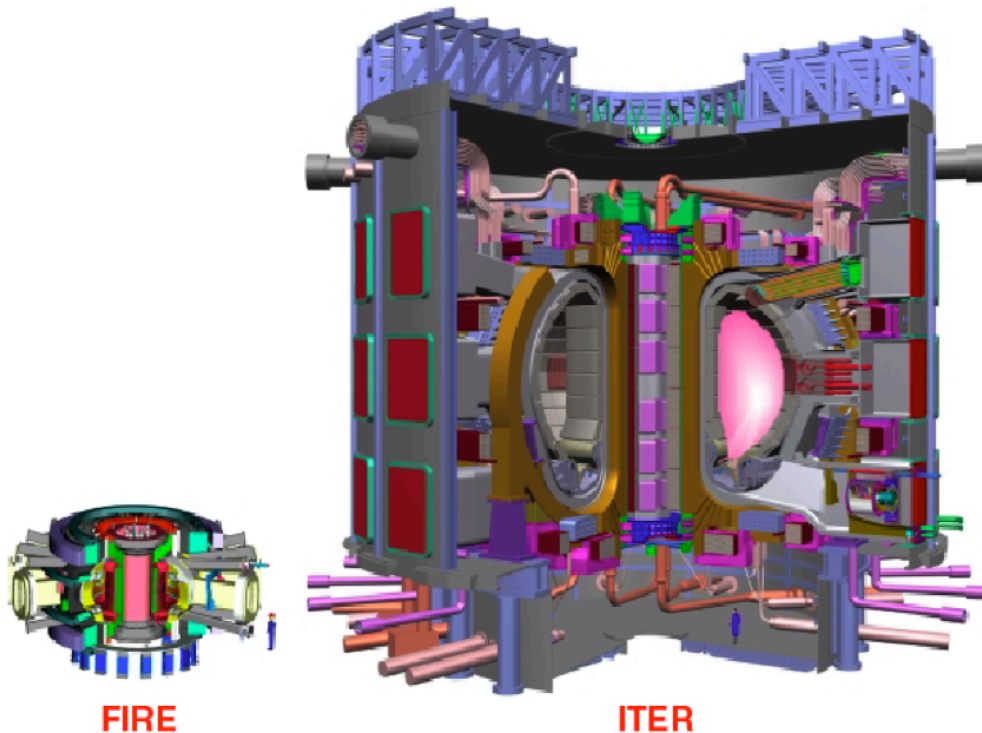
Nucl. Fusion **47** (2007)
Huguet, FED **36**(1997)

Different magnet concepts: complex tradeoffs in available B, P_{coil} , operating T & pulse duration

Concept	B_{max} (T)	T (K)
Water-cooled Cu	10	300
Cryogenic Cu	22	70
Nb-Ti Superconductor	7	4
Nb-Sn Superconductor	12	4
REBCO Superconductor		



NAS study: Cryogenic Cu could study burning plasma science at 25x smaller volume than Nb₃Sn



$$p_{th} \tau_E \sim R^{2.7} B^{5.5}$$

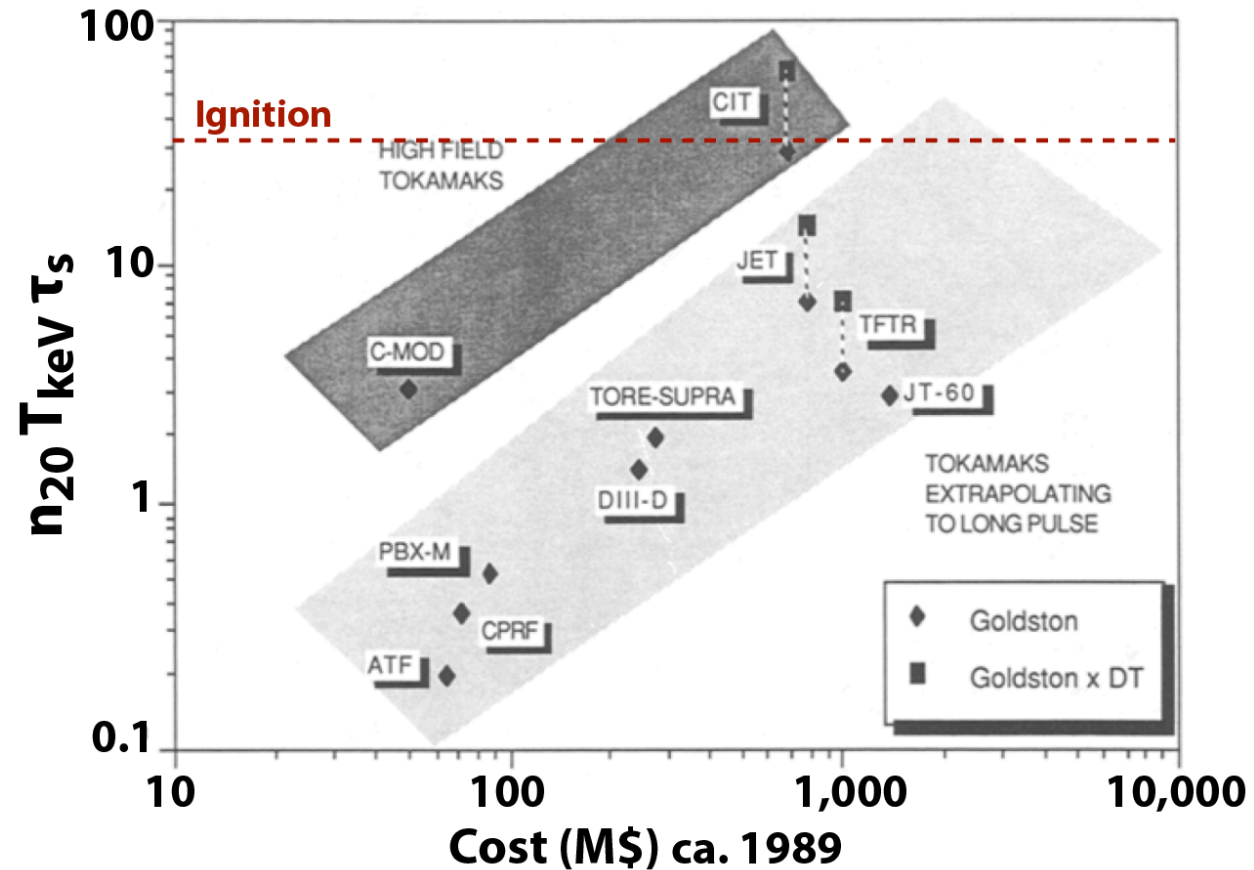
$$\text{Volume} \sim R^3 \sim 1/B^5$$

	FIRE	ITER
B (T)	10	5.3
R (m)	2.14	6.2
Q	10	10
τ / τ_{CR}	> 1	> 1
V_p (m³)	30	800

→
25x

Tactics? High-B, compact was known to have ~10-fold performance to cost ca. 1990 but pulsed

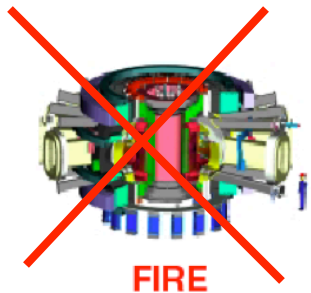
Compact Tokamak
Ignition Concepts
J. Willis
J. Fusion Energy **1989**



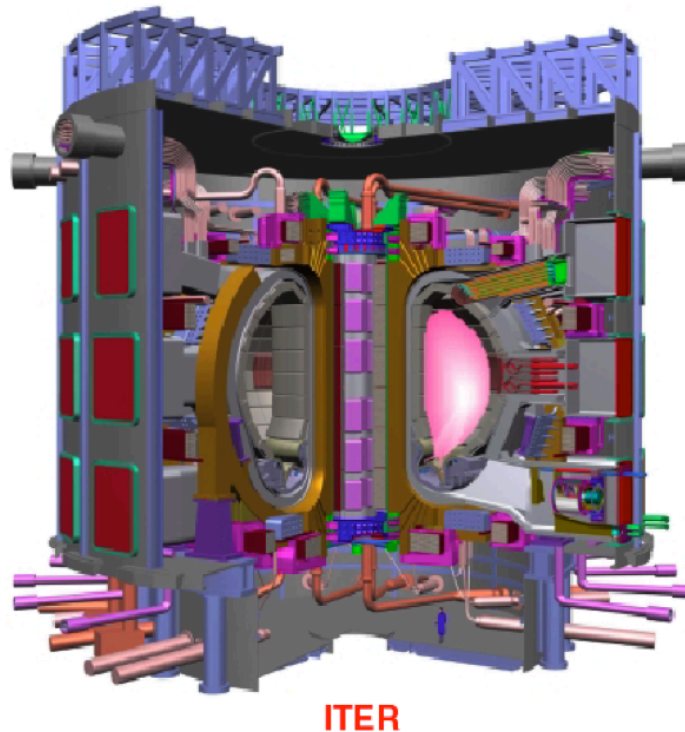
Cryogenic Cu, embodied in FIRE, could study burning plasma science at 25x smaller volume

$$p_{th} \tau_E \sim R^{2.7} B^{5.5}$$

$$\text{Volume} \sim R^3 \sim 1/B^5$$



“Dead-end Technology”

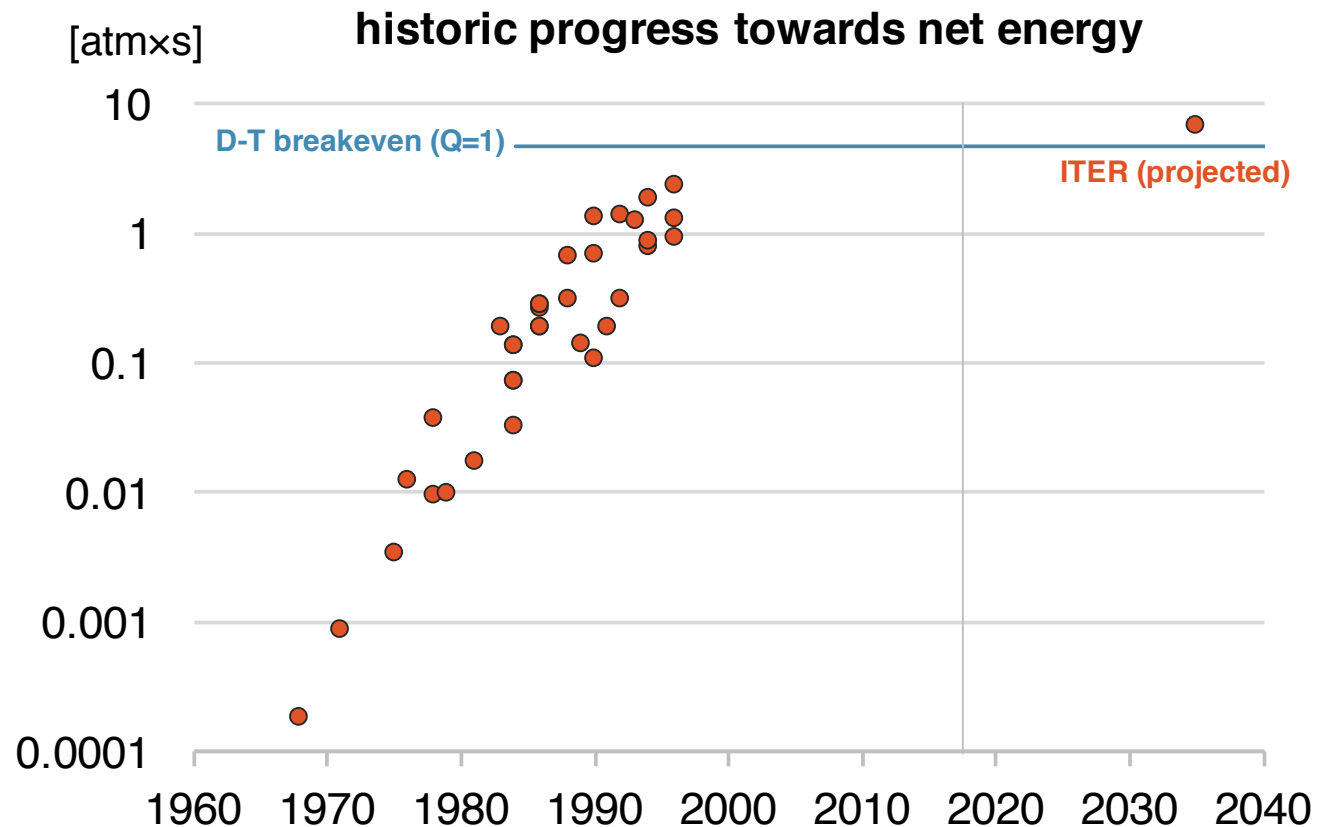


	FIRE	ITER
B (T)	10	5.3
R (m)	2.14	6.2
Q	10	10
τ / τ_{CR}	> 1	> 1
V_p (m³)	30	800

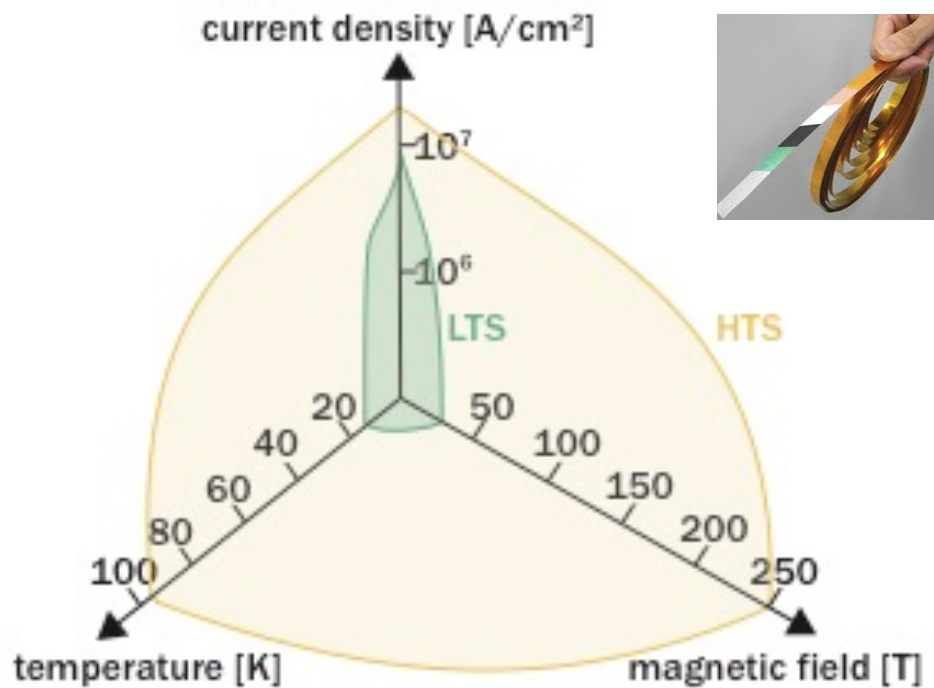
→
25x

Choosing ONLY the low-field, large volume path now appears to have severe tactical disadvantages

- 40-year gap!
- A single experiment in the entire world!
- Superconductor limits changed meanwhile!



High-Temperature (HTS) REBCO superconductors

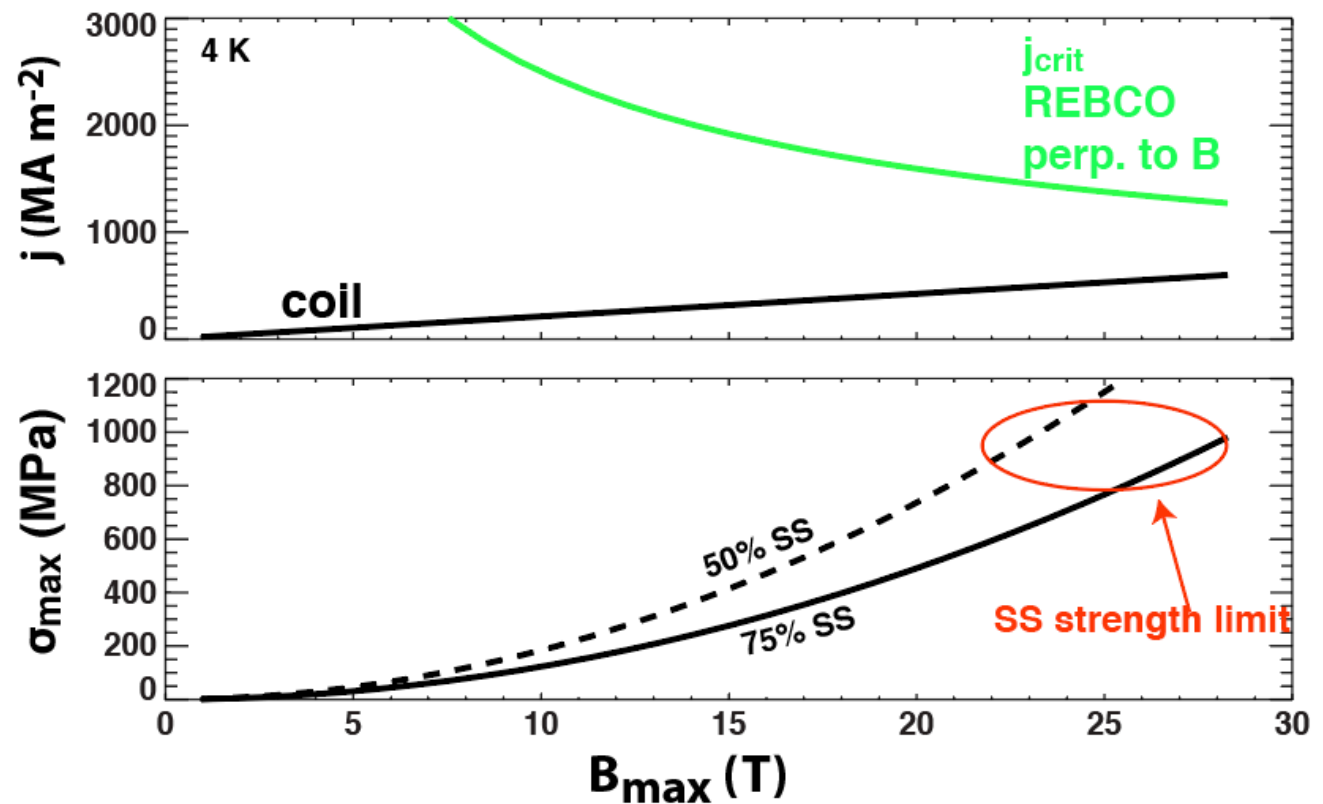
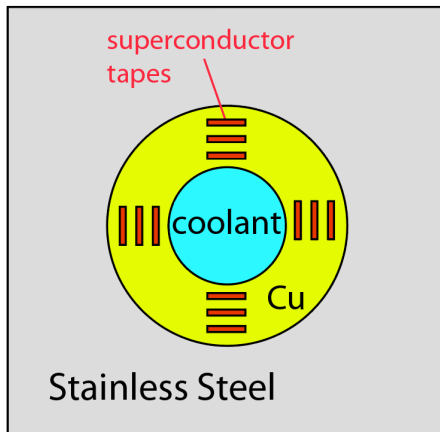


$$\frac{J_c}{J_{c,0}} = \left(\frac{B}{B_0} \right)^{-\alpha}$$

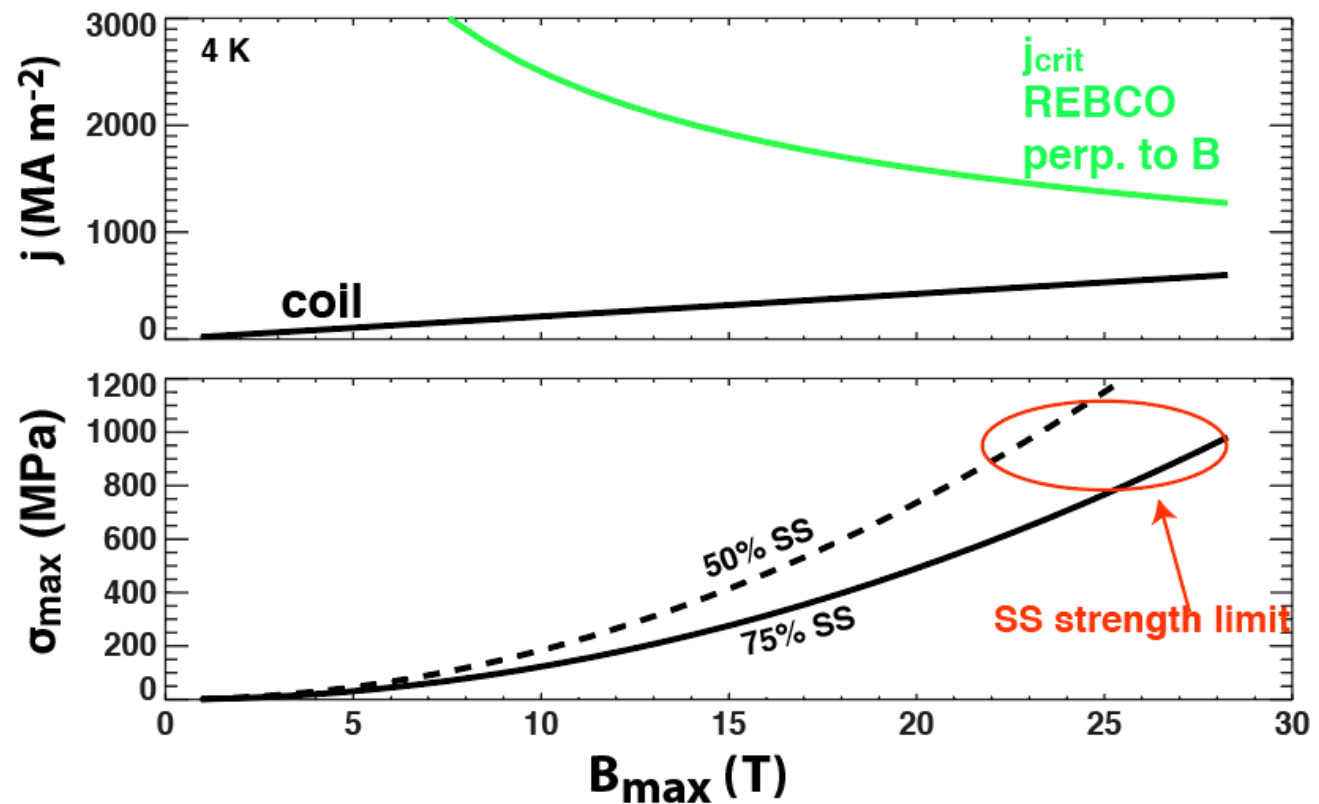
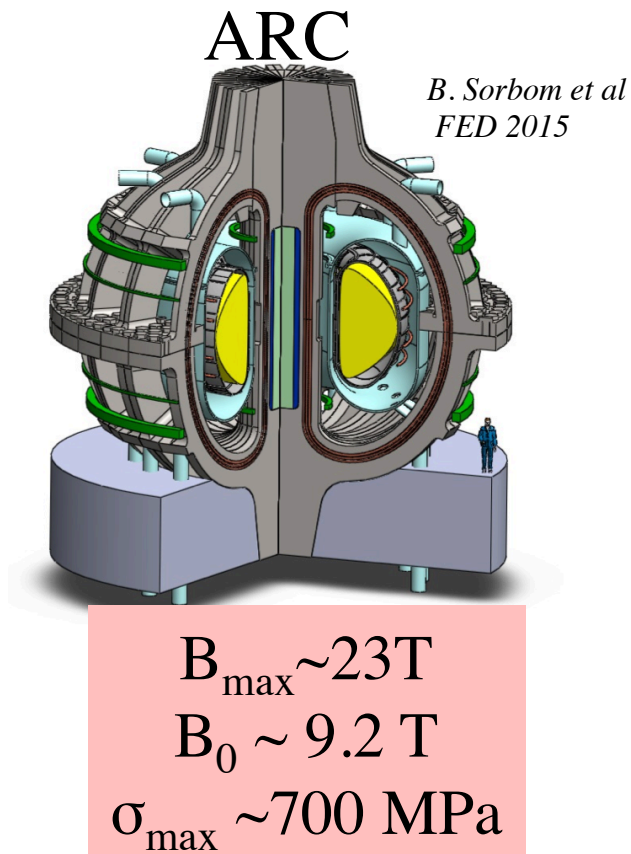
	J_{c0}	B_0	α
Nb-Ti	10^3	5	3
Nb ₃ -Sn	10^3	10	3
REBCO	2.5×10^3	5	0.6

$$T \sim 4 \text{ K}, B > B_0$$

With HTS magnets, stress is the only limit → multiple design choices to achieve $B_0 > 10\text{T}$



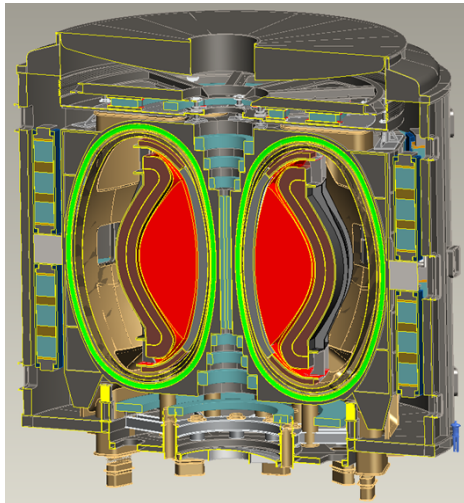
With HTS magnets, stress is the only limit → multiple design choices to achieve $B_{\max} > 20\text{T}$



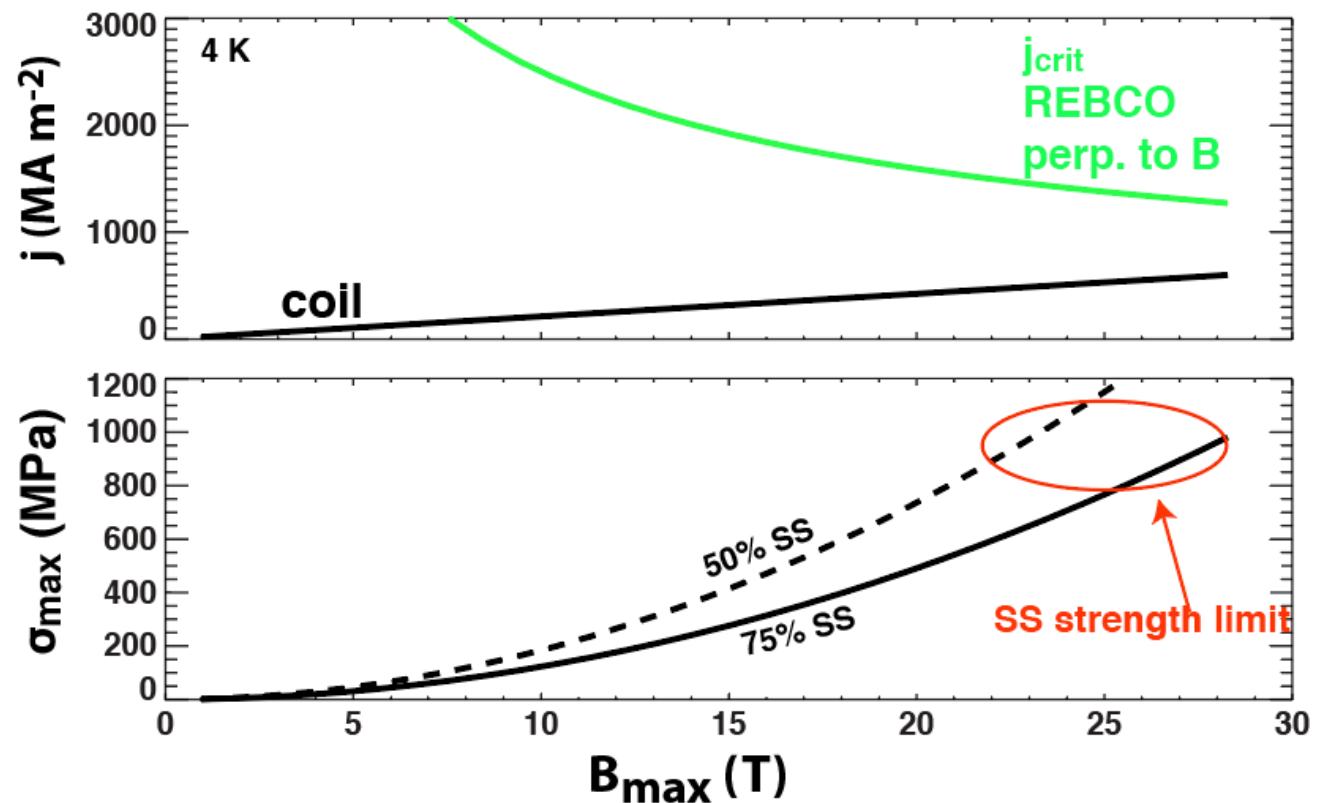
With HTS magnets, stress is the only limit → multiple design choices to achieve $B_{\max} > 20\text{T}$

Pilot ST

Menard et al Nucl. Fusion 56 2017

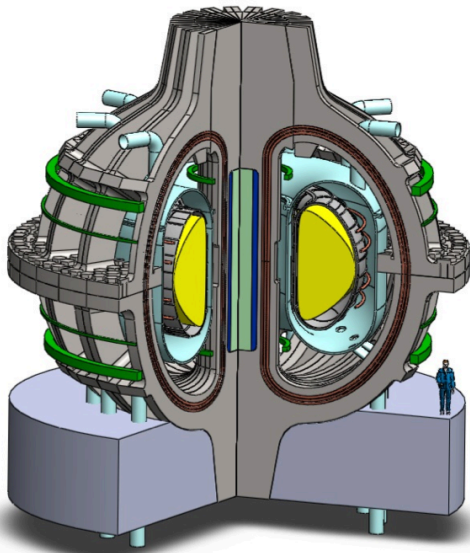


$B_{\max} \sim 19\text{ T}$
 $B_0 \sim 6\text{ T}$

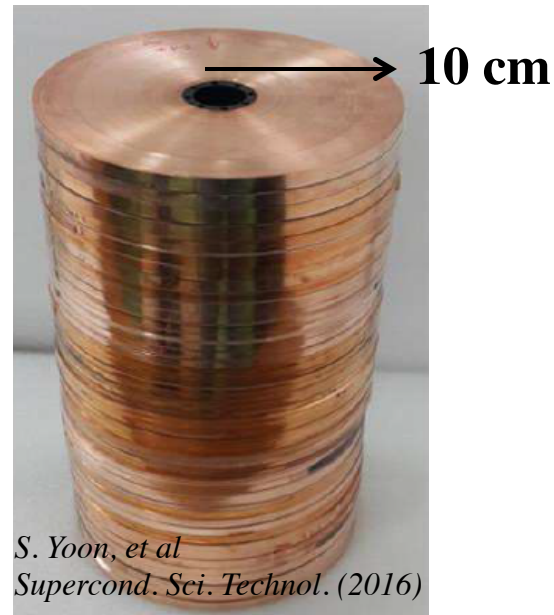


With HTS magnets, stress is the only limit → multiple design choices to achieve $B_{\max} > 20\text{T}$

ARC



$$B_{\max} \sim 23\text{T}$$
$$B_0 \sim 9.2\text{ T}$$

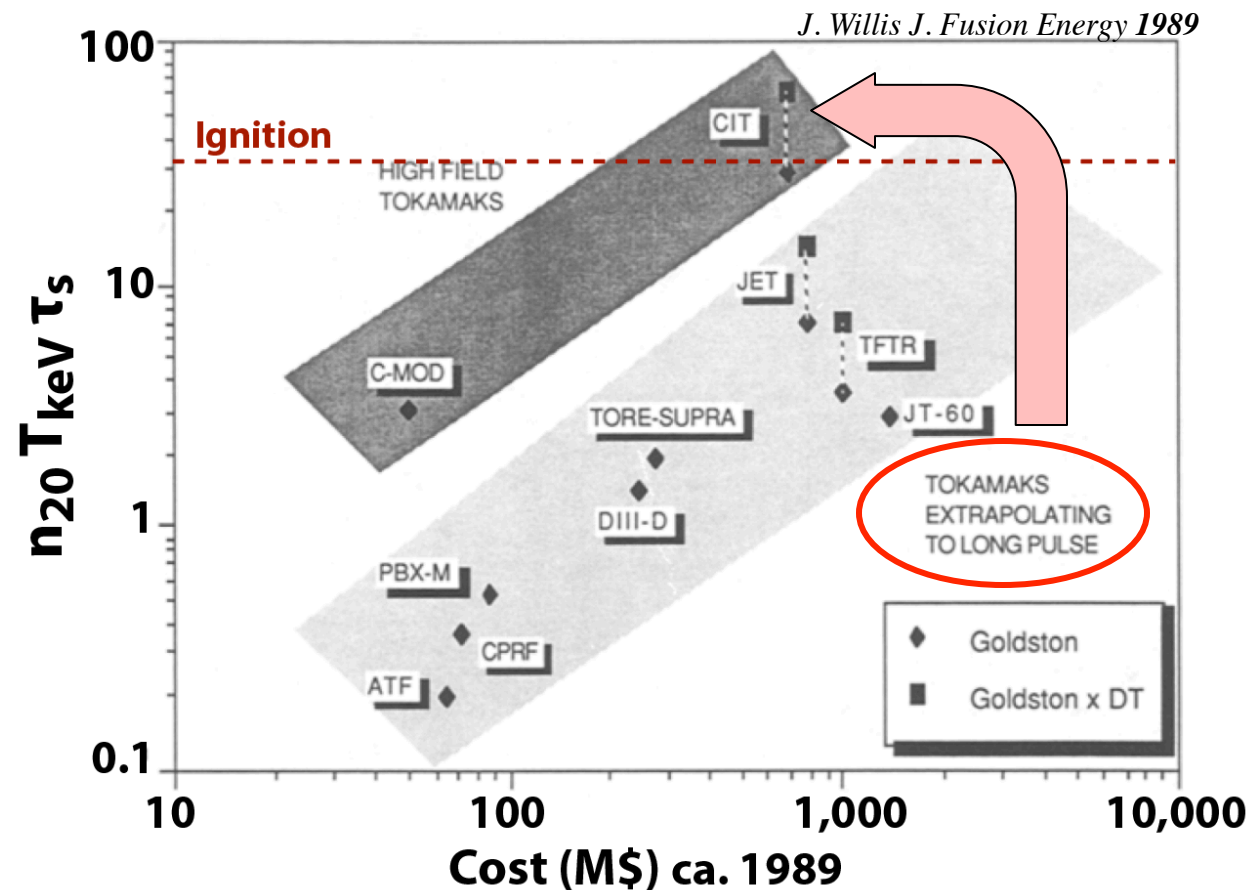


S. Yoon, et al
Supercond. Sci. Technol. (2016)

Small NMR magnets **$B > 45\text{ T}$**
Already demonstrated

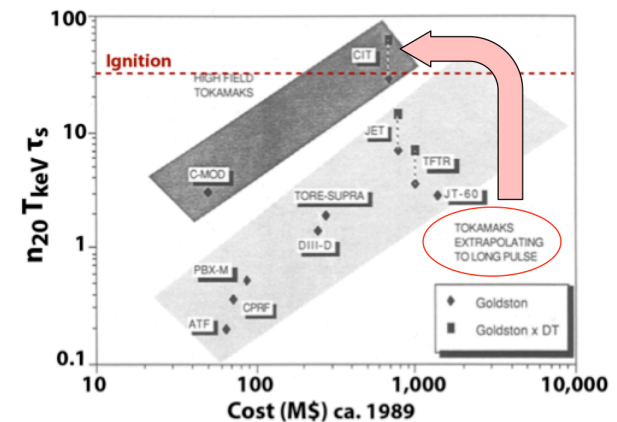
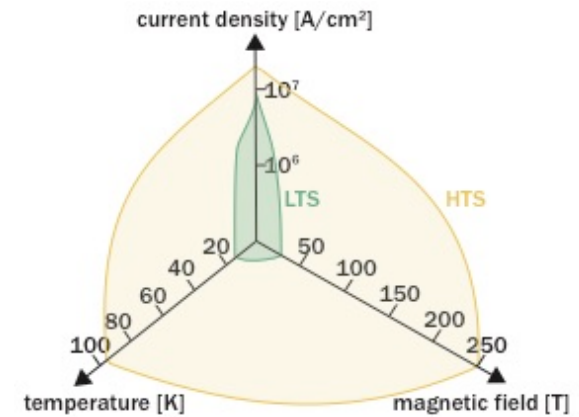
HTS magnets clearly change the tactical landscape for magnetic fusion

Concept	$B_{\max}$ (T)	T (K)
Water-cooled Cu	10	300
Cryogenic Cu	22	70
Nb-Ti Superconductor	7	4
Nb-Sn Superconductor	12	4
REBCO Superconductor	>23	4 - 30



I rest my case

Issue	Scaling		Issue	Scaling	
Power density	B^4	😊	Density (tokamak)	$R^{-1} B^1$	😊
Confinement (generic)	$R^2 B^2$	😊	Density (stellarator)	$\beta B^{2.5}$ (burning)	😊
Confinement (tokamak)	$R^{2.7} B^{3.5}$ (H_{98}) $R^{3.1} B^{2.1}$ (Petty)	😊	Heat exhaust: min. f_Z	$R^{1.3} B^{0.9}$	😊
Confinement (stellarator)	$R^{2.8} B^{2.1}$	😊	Heat exhaust: $q//$	B^{-1} (burning)	🤔
Gain	$R^{2-3.1} B^{4-5.5}$	😊	Runaway e- amp.	$\exp(R^{0.28} / B^{0.3})$	😊
Stable pedestal	$\sim \beta_N B^2$	😊	Synchrotron: runaways	B^2	😊
			Synchrotron: thermal	$\sim B^{1.5}$	😊
			TAE	$n \sim B, v_A \sim B$	😊





Plasma Science and Fusion Center
Massachusetts Institute of Technology

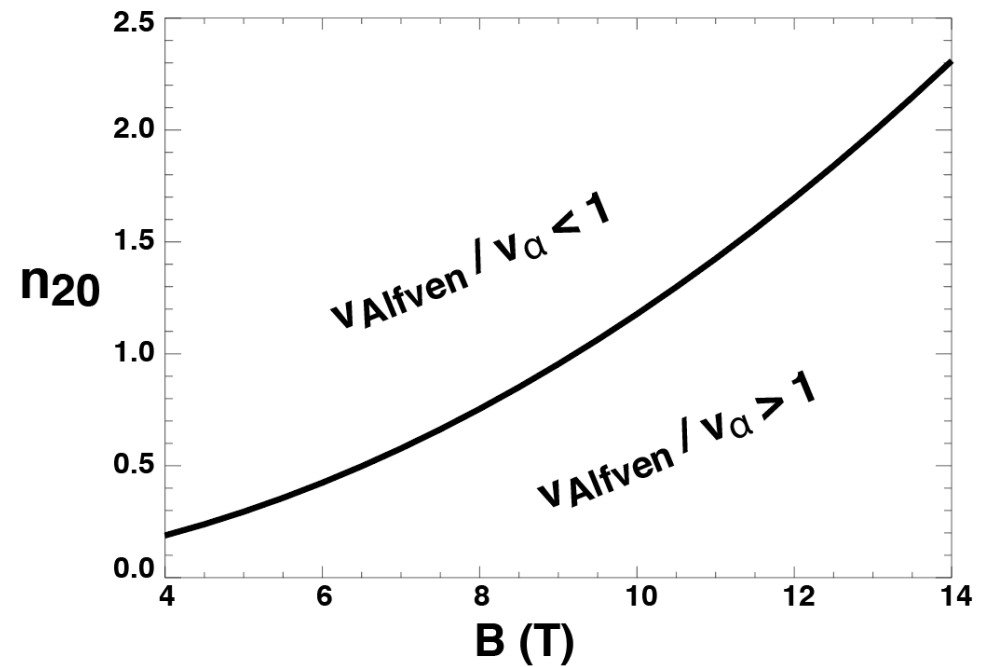


psfc.mit.edu

Collective Alfvénic instability

$$\gamma \sim \frac{\partial \beta_{\alpha}}{\partial r} \left(1 - \left[\frac{v_{\text{Alfven}}}{v_{\alpha 0}} \right]^2 \right)$$

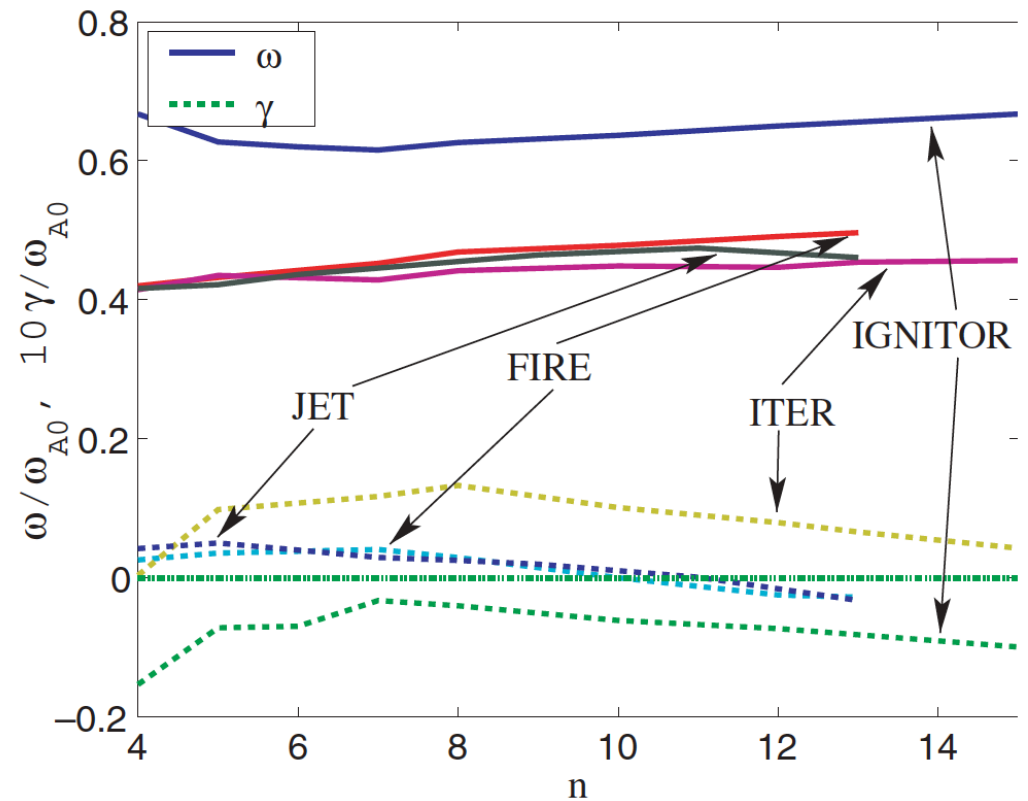
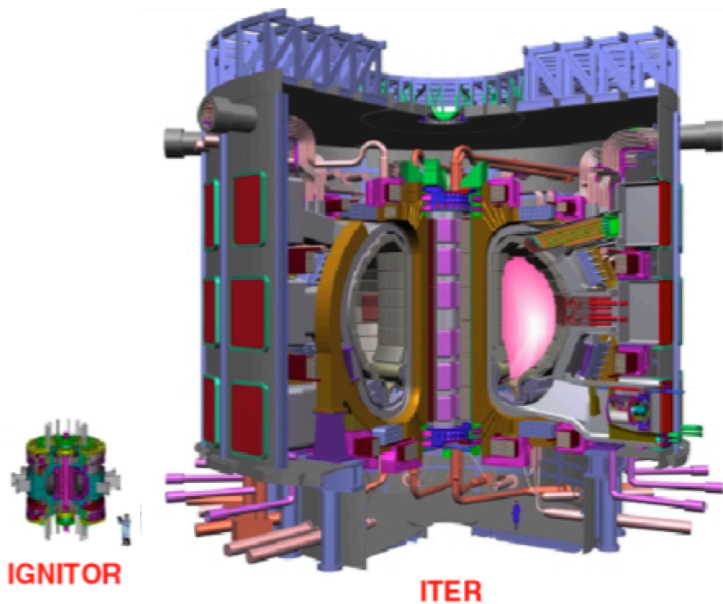
$$\frac{v_{\text{Alfven}}}{v_{\alpha,0}} \sim \frac{B}{n^{1/2}}$$



Gorelenkov et al Nucl Fusion **43** (2003)

“IGNITOR turns out to be robustly stable to TAEs”¹
High n, low T ignition → strong collisional damping,

$$\gamma \sim \frac{\partial \beta_{\alpha}}{\partial r} \left(1 - \left[\frac{v_{\text{Alfven}}}{v_{\alpha 0}} \right]^2 \right)$$

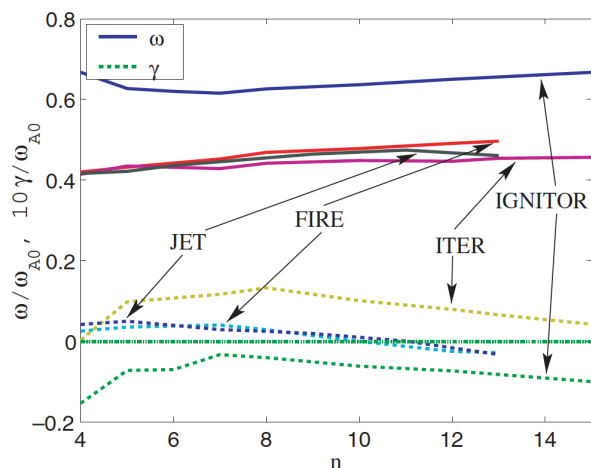


1 Gorelenkov et al Nucl Fusion **43** (2003)

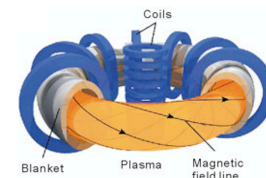
High B enables greater flexibility in optimizing n, T → synchrotron & TAE solutions

Both favor high n_{20} , lower T

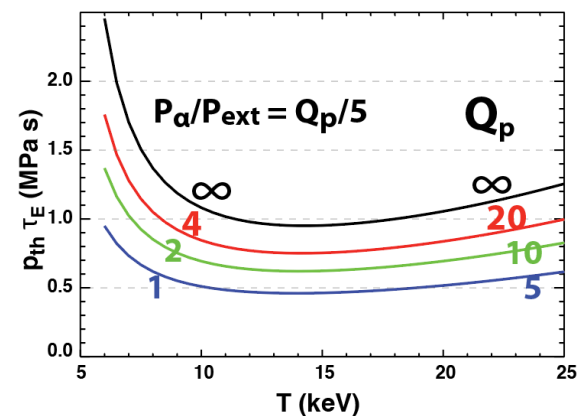
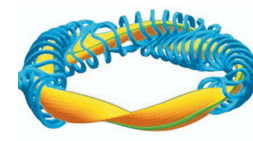
$$P_{synch} \approx 8 \times 10^{-7} \frac{n_{20} T_{keV}^{2.75} B^{1.5}}{(\beta_e R_m)^{1/2}}$$



$$n_{max} \propto \frac{B}{R}$$

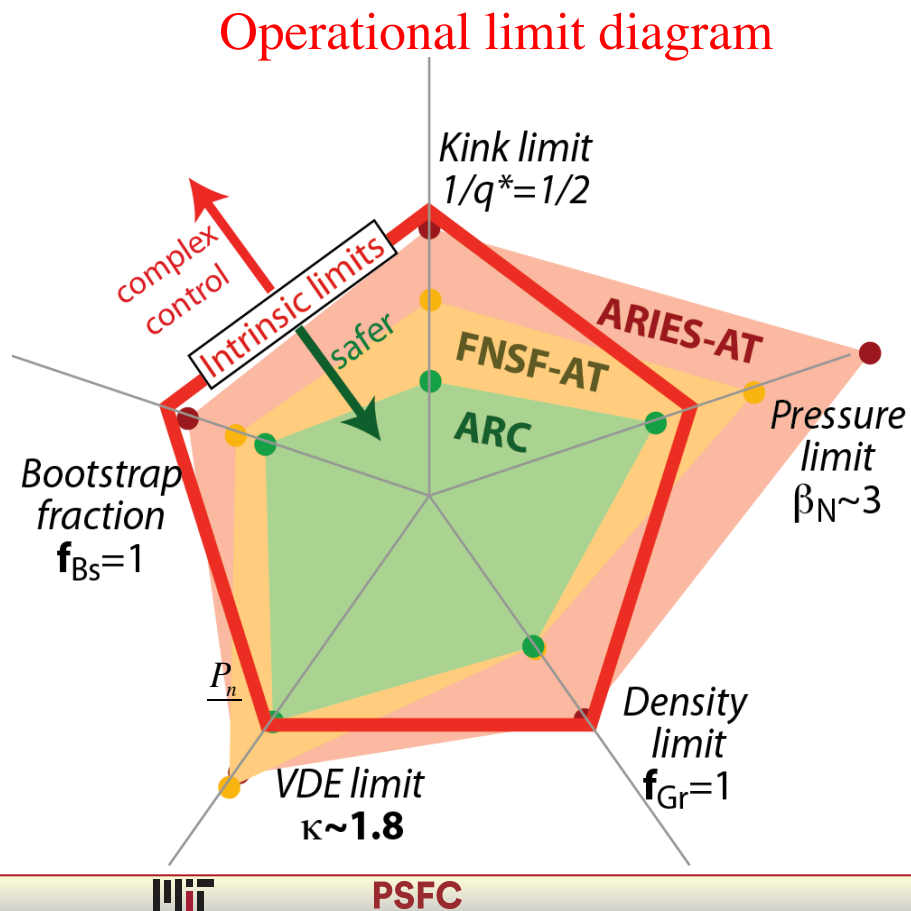


$$n_{max} \propto \beta B^{2.5}$$



ARC operating diagram

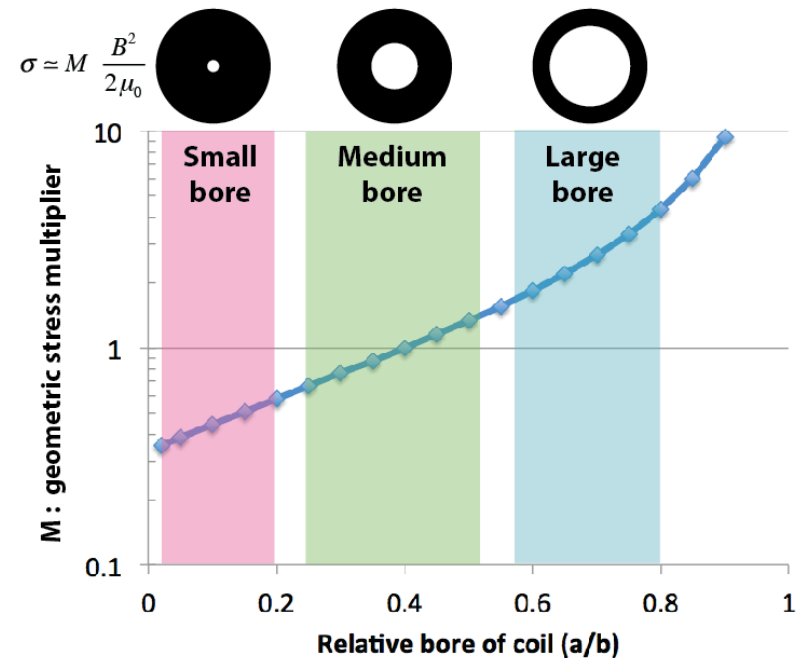
$$nT \tau_E \sim \frac{\beta_N H}{q_*^2} R^{1.3} B^3$$



As in toroidal plasma physics, aspect ratio is a critical optimization

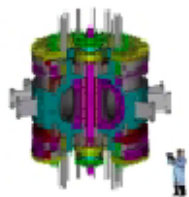
$$B_0 = 20 \text{ T} \rightarrow B^2/2\mu \sim 160 \text{ MPa}$$

	Cu	SS (304 L)
Tensile strength (MPa)	200	1340
Resistivity (nohm-m)	~2	500

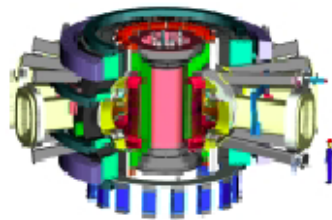


Snowmass

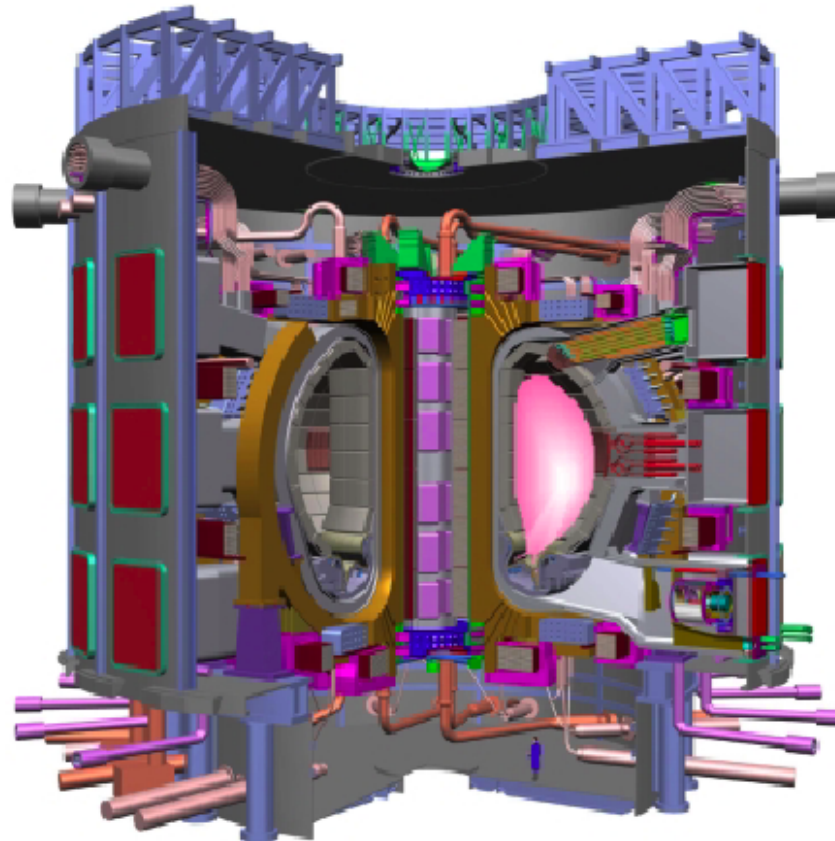
(same scale)



IGNITOR



FIRE



ITER



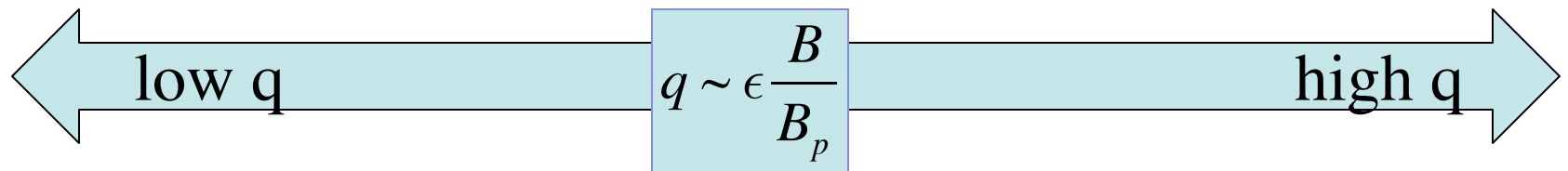
PSFC

Plasma / device optimization in physics parameters involve tradeoffs e.g. magnetic winding safety factor q for a tokamak

Benefits

- Increased τ_E

- Increased bootstrap fraction
- Less disruptive



Detriments

- More disruptive
- Low n/m modes

- Decreased τ_E

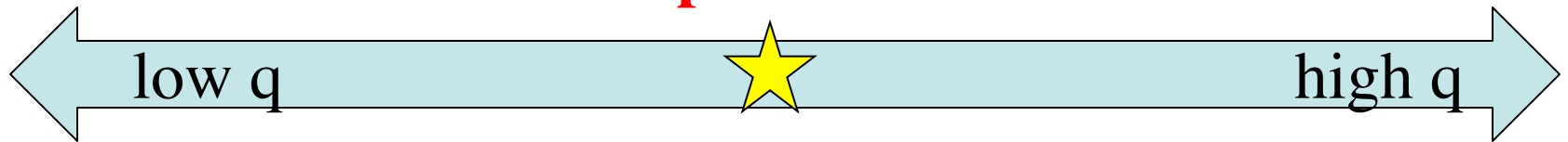
Plasma / device optimization in physics parameters involve tradeoffs e.g. magnetic winding safety factor q for a tokamak

Benefits

- Increased τ_E

- Increased bootstrap fraction
- Less disruptive

Optimum?



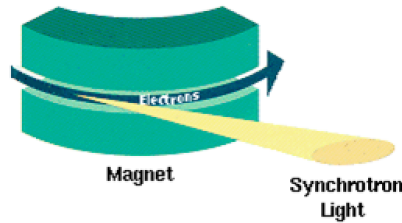
Detriments

- More disruptive
- Low n/m modes

- Decreased τ_E

Synchrotron radiation

$$P_{\text{synchrotron}} \sim \frac{E^2 B^2}{m^4}$$



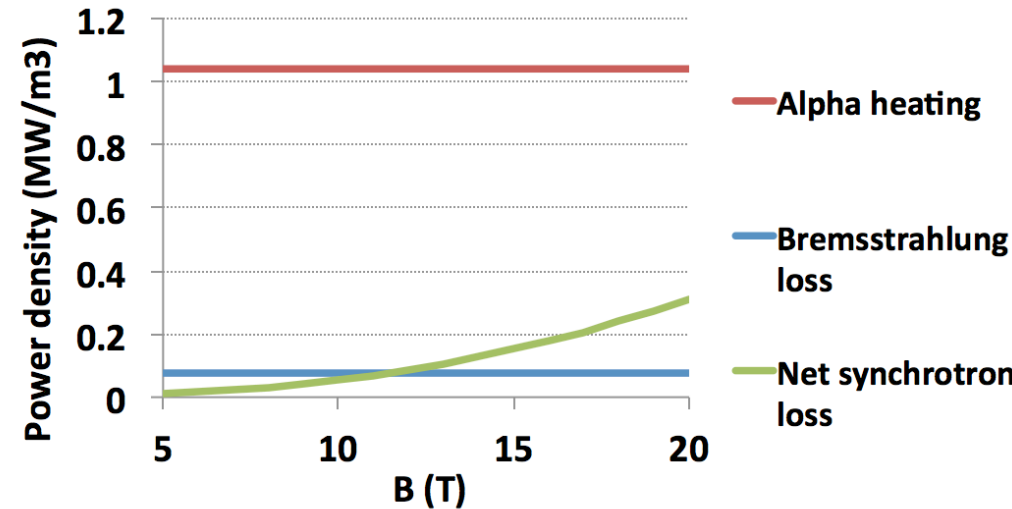
Following Rosenbluth's treatment

$$P_{\text{synch}} \approx 8 \times 10^{-7} \frac{n_{20} T_{\text{keV}}^{2.75} B^{1.5}}{(\beta_e R_m)^{1/2}}$$

$$P_{\text{brem}} \sim 0.005 n_{20}^2 T_{\text{keV}}^{1/2} [MW \text{ m}^{-3}]$$

$$P_{\text{synchrotron}} \approx 8 \times 10^{-7} \frac{n_{20} T_{\text{keV}}^{11/4} B_T^{3/2}}{(\beta_e R)^{1/2}} [MW / m^3]$$

with R in meters.



$$L \approx 2 \times 10^{-17} \frac{n_{20} T_{\text{keV}}^{11/4} B^{3/2}}{\beta_e (\beta_e R B)^{1/2}} [keV / s]$$

$$L \sim \frac{n T^{15/4}}{\beta (\beta_e R B)^{1/2}} \sim \frac{(n T) T^{11/4} B^2}{(n T) (\beta_e R B)^{1/2}} \sim \frac{T^{11/4} B^{1.5}}{(\beta_e R)^{1/2}}$$



Bohm + Gyro-Bohm confinement (generic)

$$\chi_{Bohm} \sim \frac{T}{B}$$
$$\tau_{Bohm} \sim \frac{a^2}{\chi_{Bohm}} \sim \frac{a^2 B}{T}$$

$$\chi \sim \rho_*^{\alpha_p} \chi_{Bohm} \sim \frac{\rho_*^{\alpha_p}}{B}$$

$$\tau \sim \frac{a^2}{\chi} \sim a^2 B \rho_*^{-\alpha_p}$$

$$B\tau \sim a^2 B^2 \rho_*^{-\alpha_p} \sim \rho_*^{-2} \rho_*^{-\alpha_p} \sim \rho_*^{-(2+\alpha_p)}$$

Slide Title

- **Point**
- **2nd point**